CAPABILITY, COMPETITION AND SOCIAL INTERACTIONS IN SERVICE OPERATIONS

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ABSTRACT

Rapid development of information technology leads to the current *networked and social economy*, where *social interactions* become increasingly prevalent and influential in the consumer market. Through various popular social media, consumers can easily access any information about particular products or services, where recommendations, opinions, and choices of others significantly influence their purchase decisions. The influence of social interactions brings not only opportunities for firms to survive and succeed, but also challenges in operations management in the fiercely competitive market.

Under social interactions, firms should be equipped with fitting operational capabilities to achieve profitability, market share expansion, product and brand recognition as well as customer loyalty enhancement. From the practical point of view, *how operational decisions should be adapted to the changes of consumer purchase behaviors* is a critical challenge for managers to tackle in order to better leverage the influence of social interactions. From the academic perspective,*the impact of social interactions on operational decisions and its managerial implications* is worth investigation. Therefore, in this dissertation, we focus on operational decisions including capacity, price and quality under social interactions.

We first investigate capacity/service rate and price decisions in a fixed size consumer market under competition with two homogeneous firms. Although social interactions can always benefit a monopolistic firm, under competition, firms can benefit from social interactions depending on market size and social interaction intensity. We demonstrate that a small market or a higher social interaction intensity will lead to a higher service rate or a lower price which reduces firm profit; while consumers may always benefit from social interactions with positive expected surplus. If the market size is large or the social interaction intensity is low, firms may still get benefit from social interactions, where price and service rate can be determined to extract all consumer expected surplus.

Next, we focus on how managers can better leverage social interactions in operations to increase profitability, by focusing on the price and capacity decisions of a monopolistic firm in a market with repeated interactions. The main result indicates that under social interactions, strategic pricing or capacity policies always induce a monotonic arrival rate path, which converges to a unique steady state. A lower price

or a higher capacity decision is desired to build up a larger customer base which will induce more potential consumers under social interactions. If price and capacity are simultaneously determined, a lower profit margin may be expected to increase the customer base. Strategic policies can always achieve a larger profit after initial periods when a larger market base has been established due to social interactions.

We then investigate the impact of social interactions on resource investment decisions on service quality, termed as service effort decisions for a profit maximizing firm. Under social interactions, particularly the word-of-mouth communication, heterogeneous consumers are backward-looking and adaptive in purchase decision-making. Service effort decisions impact both post-purchase experiences pre-purchase expectations which affect consumer satisfaction. It is found that optimal service effort policies always induce monotonic overall experience paths which converge to a unique steady state level. Social interactions always lead to a higher steady state service effort level, especially due to the negative social interaction effect. Under satisfaction-dependent social interactions, a constant service effort policy may be optimal to offer consistent quality of service, especially when consumers have high initial expectations.

This dissertation adopts multiple research methodologies to build analytical models to investigate operational decisions under social interactions, including game theoretical analysis, queueing theory, and dynamic programming techniques. Based on the analytical results, the contribution of this dissertation lies in offering fresh insights and better understanding on the impact of social interactions in operational decisions and performance, and opening new avenues for future research in operations management under social interactions.

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1. INTRODUCTION

1.1 Research Background

Rapid development of information technology and wide availability of internet connection have lead to the current networked and social economy. In the consumer market, due to various vastly emerging and increasingly popular social media, such as social networking sites, instant messengers and on-line virtual communities, consumption styles and customer purchase behavior have been influenced and changed significantly. Nowadays, consumers are influenced more and more by others through *social interactions* when making consumption decisions, since they can easily access any information about goods or services they intend to buy. A recent survey by McKinsey Global Institute reveals that, 80% of the current 1.5 billion social networking users interact with social networks regularly, while 1/3 of consumer spending could be influenced by social shopping (Chiu et al., 2012b). The increasing influence of social interactions in consumer decision making is not only prevailing in mature markets but also growing in emerging economies, like China, where 66% of Chinese consumers rely on recommendations from friends and family (Chiu et al., 2012a).

It has been observed for several decades in various disciplines that due to social interactions, decisions made by individuals are influenced by their observations, perceptions, or anticipations of decisions made by others (Cialdini, 2001; Lopez-Pintado and Watts, 2008). Pioneered by Becker (1974), the impact of social interactions on individual social and economic behaviors, such as investment in education, crime in a neighborhood (Glaeser et al., 1996), drug abuse, political opinion formation, etc. in a society (Brock and Durlauf (2001), Lopez-Pintado and Watts (2008)), as well as consumption choices, diffusion of products and innovations, etc. in consumer markets, has been studied for several decades. Scheinkman (2004) defines social interactions as the *"particular forms of externalities, in which the actions of a reference group act on an individual's preferences."* Due to social interactions, "*the utility or payoff an individual receives from a given action depends directly on the choices of others in that individual's reference group*" (Brock and Durlauf, 2001), and individual actions may affect the *constraints, expectations,* and/or*preferences* of others (Manski, 2000).

Many factors or underlying forces that drive individuals to be susceptible to social influences from interactions with others when making social and economic choices have been identified and analyzed. Summarized by Lopez-Pintado and Watts (2008), individuals may desire to be recognized within certain social groups (Festinger et al., 1950), to be differentiated from others (Simmel, 1957), to avoid anticipated sanctions due to peer pressure by conforming to group behavior (Kandel and Lazear, 1992), or as a socially conditioned response to authority (Milgram, 1969); following the choices of others may serve as a means of reducing the complexity of decision-making processes (Gigerenzer et al., 1999). Due to information asymmetry when making choices, individuals may herd as a way of inferring otherwise inaccessible information (Scharfstein and Stein, 1990; Banerjee, 1992; Bikhchandani et al., 1992, 1998; Goldstein and Gigerenzer, 2002). For some products or services, such as the telecommunication service, due to *network externalities* (Katz and Shapiro, 1985, 1994), the utility of the product or service to an individual is positively related to the network size which drives consumers to purchase the same brand.

Social interactions among individuals result in the interdependence between individual decision-makers, which in turn affect the *collective behavior* of the corresponding population dramatically in an aggregate level. Theoretical and empirical studies have documented various social and economic phenomena due to the influence of social interactions on individual decision-making, such as the sudden change of a neighborhood from integrated to segregated (Schelling, 1971), the popularity of books, songs and movies from nowhere to become a hit (Salganik et al., 2006), and the markets locked in to inferior technology (Arthur and Lane, 1993), the herding behavior in the financial market (Avery and Zemsky, 1998; Bikhchandani and Sharma, 2001) and online market (Onnela and Reed-Tsochas, 2010).

The influence of social interactions in consumer purchase decisions is also prevalent, which has been widely investigated. Due to social interactions, consumers are influenced substantially by the *reference group effect* (Bearden and Etzel, 1982), *observational learning* (OL) (Cai et al., 2009), *word-of-mouth* communication (WOM), etc. in purchase decision-making. Three reference group effects have been identified, namely *informational, utilitarian*, and *value-expressive* influence, where a reference group is "*a person or group of people that significantly influences an individual's behavior*" (Park and Lessig, 1977). Informational influence is due to uncertainty about the product or service, such as the quality. Utilitarian influence refers to the attempts to comply with the wishes of others to achieve rewards or avoid punishments. Value-expressive influence is characterized by the need for psychological association with a person or group and is reflected in the acceptance of positions expressed by others. As a particular way of social interactions, WOM is "*the primary factor behind 20% to 50% of all purchase decisions*" and the influence is "*greatest when customers are buying a product for the first time or when products are relatively expensive*" (Bughin et al., 2010).

Two categories of models that incorporate social interactions in consumer choices, namely the "heuristic" models and "utility" models (Lopez-Pintado and Watts, 2008), have been developed. Heuristic models include the diffusion models of innovations and collective actions, such as those developed in Anderson and May (1991), Glaeser et al. (1996), Granovetter (1978), and Watts (2002). In those models, individuals adopt simple rules as a function of previous decisions of other actors. In marketing research, the Bass model of diffusion (Bass, 1969) belongs to the category of heuristic models. Relying on utility functions to describe individual preferences, utility models incorporate social influences directly based on assumptions about the psychological or economic details of the human decision-making process. Herding models (Banerjee, 1992; Bikhchandani et al., 1992, 1998) due to incomplete information and consumer observational learning behaviors are typical examples. The anti-coordination games (Bramoullé, 2007), and coordination games (Ellison, 1993; Kandori et al., 1993; Morris, 2000; Young, 1998) are other typical examples of utility models.

The prevailing social media facilitates social interactions among consumers substantially and amplifies and accelerates the influence in consumer purchase behaviors to a great extent. Social interactions bring not only opportunities for firms to expand market share, achieve stable growth and increase profitability, but also challenges operations management to better serve increasingly demanding consumers. Taking WOM as an example, "$\cdots$ *word-of-mouth is no longer an act of intimate, one-on-one communication. Today, it also operates on a one-to-many basis* $\cdots$. *Some customers even create web sites or blogs to praise or punish brands*" (Bughin et al., 2010). However, "*few have a deep understanding of exactly how social media interacts with consumers*" (Divol et al., 2012), although companies are investing substantial resources into social media. Therefore, better understanding of the influence of social interactions in consumer purchase decisions is critical in operations. From a managerial perspective, how operational decisions are impacted under social interactions is of practical significance to improve decision-making and firm performance.

The influence of social interactions can be analyzed along consumer decision-making processes. Typically, consumers make purchase decisions through four primary phases: initial consideration, active evaluation and comparison, selection and purchase, and post-purchase experience (Court et al., 2009). In the initial consideration stage, a substantial proportion of potential brands may be considered through social interactions. In the evaluation and comparison stage, consumers now rely more on information through social interactions to make purchase decisions, such as online product reviews and recommendations from others. During transaction processes, how products or services are delivered during service encounters influences consumer purchasing experiences substantially. After purchasing, consumer satisfaction and loyalty will be determined. They become more and more active in sharing experiences which will influence potential consumer purchase decisions. Therefore, social interactions become critical in the cycle of the consumer decision process.

Although social interactions become significant in consumer purchase decisions, there is a lack of research on how operational decisions and firm performance would be impacted under social interactions. Previous research either focuses on the underlying factors that drive the influence of social interactions in individual decision making or studies the dynamics of the collective behavior under social interactions. Very few studies focus on the impact of social interactions on firm performance from an operational perspective. Motivated by academic needs and practical utilities, in this dissertation, we focus on how managers can better improve operational decision-making to enhance firm performance and customer satisfaction under social interactions. Specifically, under the influence of social interactions in consumer purchase decisions, the following three topics are investigated:

1. The impact of social interactions on the price and capacity decisions in a competitive market (Chapter 2);

2. The impact of social interactions on the price and capacity decisions in the long-term operations (Chapter 3);

3. The impact of social interactions on the service effort decision in product/service delivery (Chapter 4).

The operational decisions investigated are capacity/service rate, price as well as service quality. Critical to the success of any profit-making company, we investigate how social interactions would impact operational decisions in terms of capacity, price, and quality in profit optimization, market expansion and consumer satisfaction enhancement.

1.2 Dissertation Structure

Drawing on extensive studies on social interactions, operations management, and consumer behavioral theories, the above three research topics are investigated through analytical models. Fig.1.2.1 illustrates the framework of this dissertation.

In Chapter 2, the impact of social interactions on the price and capacity decisions in a competitive market is investigated, where two homogeneous firms compete for market share with a fixed market size. The model is a static model, where consumers have rational expectations when making purchase decisions (or strategic queue-joining decisions), and operational decisions are derived based on the rational expectation framework (Katz and Shapiro, 1985). Through analyzing the impact of social interactions and the interplay with competition on operational decisions,

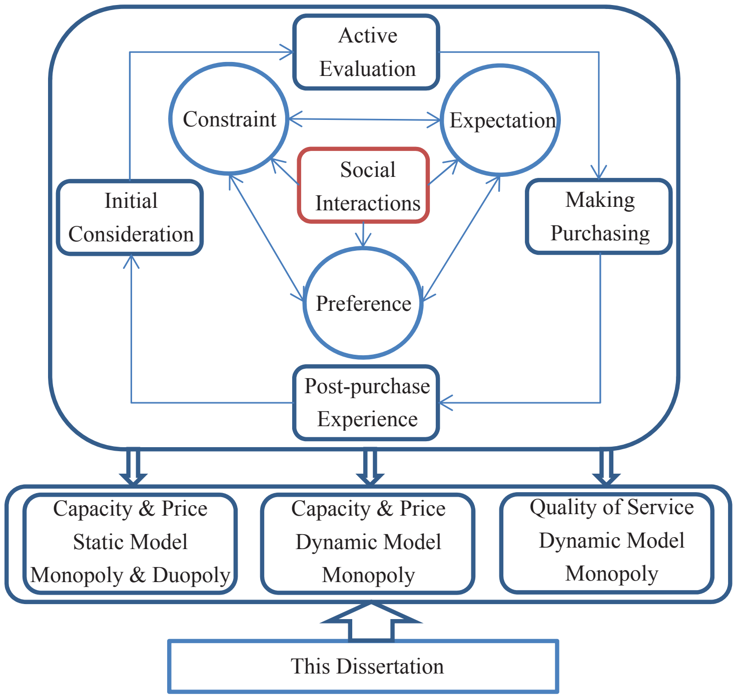

Fig. 1.2.1: Dissertation structure.

this chapter contributes to the research in operations management, particularly, the competition in a service system from a game theoretical perspective. The impact of social interactions on the price and capacity decisions in the long-term operations is investigated in Chapter 3, where a monopolist firm maximizes its profit through price and capacity decisions in the consumer market with repeated interactions. The firms in Chapter 2 and Chapter 3 are operated as M/M/1 queueing systems to capture congestion due to capacity constraint. Social interactions are captured by the phenomenon that consumers perceive a higher value of a product or service when more consumers make the same choice (Chapter 2) or the existing market share or customer base (Chapter 3). Consumer choices under social interactions are developed based on heuristic models, where a social utility term is incorporated in consumer purchase decisions. By investigating both positive and negative externalities due to social interactions in a service system, Chapter 3 contributes to the study in operations management, particularly how operational decisions should be adapted in the new economy. The impact of social interactions on the service effort decision is studied in Chapter 4, where a profit-maximizing firm needs to decide how much resource should be invested (service effort) to provide a certain quality of service in product delivery or service encounters. Service effort decisions impact both experiences and expectations, which determine consumer satisfaction. Social interactions, particularly the word-of-mouth communication, influence consumer purchase decisions. The contribution of this chapter lies in offering new insights on the impact of social interactions on service operations management.

Each topic captures the three main interactions among consumers in purchase decision-making, namely the constraint, expectation and preference (Manski, 2000). In particular, the models in Chapter 2 and Chapter 3 capture the interactions on constraint and preference among consumers, where social interactions bring both positive consumption externalities and congestion effect; the model in chapter 4 focuses on the interactions among preferences and expectations, where potential consumer expectations are influenced by existing consumers. In this dissertation, social interactions play a role along all consumer decision processes. Specifically, Chapter 2 and Chapter 3 focus on how consumers make initial considerations of potential options, active evaluations of each choice and purchase decision; Chapter 4 focuses on how pre-purchase expectations and post-purchase experiences impact consumer purchase decisions. Among these three topics, the impact of social interactions on operational decisions of price, capacity and quality is investigated and managerial insights are offered based on analytical results. Conclusions, limitations, and future research directions are provided in Chapter 5.

2. SERVICE COMPETITION UNDER SOCIAL INTERACTIONS

2.1 Abstract

Consumer purchase decisions are highly influenced by the choices of others through social interactions, where they perceive a higher value of a product or service when more consumers make the same choice. Social interactions lead to more intense competition in the consumer market and bring both opportunities and challenges in operations management. Under social interactions, how operational decisions such as capacity and price are made is critical for firms to expand market share and increase profitability in the fiercely competitive market. In this chapter, we investigate the price and capacity decisions under social interactions in a fixed size consumer market between two competing homogeneous firms which are operated as two M/M/1 queueing systems. The capacity of each firm is determined by the service rate, which is costly to expand. The market size is captured by the total potential arrival rate. Consumers are strategic in queue-joining decisions.

The result indicates that social interactions can always benefit a monopolist firm where a higher price is charged and a smaller capacity is determined. However, under competition, the impact of social interactions on equilibrium operational decisions depends on the market size and the social interaction intensity. A small market size or a higher social interaction intensity will lead to a higher service rate or a lower price which reduces firm profit; while consumers may always benefit from social interactions with positive expected surplus. If the market is large or the social interaction intensity is low, firms can still benefit from social interactions, where appropriate price and service rate can be determined to extract all consumer expected surplus in service competition. Taking social interactions and the interplay with competition into consideration, managers can better improve firm performance through operational decision-making.

2.2 Introduction

The rapid economic growth leads to increasingly intense competition among firms for both product deliveries and service offerings. Competition among firms for market share comes from both the supply side and the demand side. From the supply side, various similar and substitutable products or services are now supplied by many competing firms in the consumer market. Consumers have many alternatives when making consumption choices. Due to technological advances, product differences and service distinctions become increasingly small and even negligible, which drive firms to rely more on properly managed operations to survive and succeed in the fiercely competitive market. How to make and implement appropriate operational strategies such as price and capacity decisions becomes critical for firms to expand market share and increase profitability.

On the demand side, due to *social interactions*, competition among firms in the consumer market becomes even more intense, since consumer preferences and purchase behaviors have evolved and changed substantially. A typical phenomenon due to social interactions is that consumers become more inclined to purchase the products and services with a large sales and customer base, such as those bestsellers, star products and popular services; they are more likely to herd into longer queues where more consumers are waiting in the same line to be served (Veeraraghavan and Debo, 2009a). Social interactions become increasingly influential due to rapid development of information technology, since consumers can easily access any information about goods or services through various social medias, such as social networking sites, instant messengers, and on-line virtual communities. Thus, social interactions become more and more significant in consumer purchase decisions, which bring opportunities for firms to boost demand and achieve rapid growth.

The influential role of social interactions in consumer purchase decisions has been realized by managers, and various strategies in marketing and operations have been applied to utilize the influence to enhance firm performance, sometimes even unethically. To attract more demand through a large sales quantity as a bestseller, authors may secretly purchase their own books[1] in order to enter into the bestseller list. Firms may adopt sales inflation to show the popularity of their products, even for big companies, such as BMW[2]. Some companies may strategically create shortages by withholding available inventories, maintaining certain waiting lists or queues to signal popularity of their products or services in order to stimulate demand[3]. For many real examples about firm practices to stimulate demand through *waiting-list effect*, see Sapra et al. (2010). In the current digital era, due to information overload, consumers become increasingly skeptical about traditional company-driven advertising and marketing while increasingly prefer to make purchase decisions based on others' recommendations, reviews, and choices through social interactions (Bughin et al., 2010).

However, the influence of social interactions in consumer purchase decisions

[1] http://www.businessweek.com/stories/1995-08-06/did-dirty-tricks-create-a-best-seller
[2] http://online.wsj.com/article/SB10000872396390444042704577589511359646688.html
[3] http://online.wsj.com/article/0,,SB10657460791372700.html

also results in challenges in operations. On the one hand, due to resource scarcity, more demand attracted through social interactions may increase the congestion of a service system. The increased congestion may drive customers away, leading to potential revenue loss, where consumers may never come back for dining[4], healthcare service[5], etc. To reduce congestion requires more capacity investment. However, capacity is always costly to expand, which restricts operational flexibility. On the other hand, since consumers become more reliant on social interactions when making purchase decisions, products or services from some dominant companies may become extremely popular and widely adopted, which leads their competitors which provide substitutable products or similar services to suffer from insufficient demand and a small market. A typical example is observed in Becker (1991), where social interactions drive consumers to join a long queue outside a popular seafood restaurant, while the nearby competing restaurant still has many empty seats, although similar service is offered and price is almost identical. Thus, social interactions may intensify competition among firms. How operational decisions would be impacted by social interactions becomes significant in firm operations management in the competitive market.

Therefore, competition and social interactions as well as their interplay call for appropriate operational strategies for firms to survive and succeed in the increasingly competitive market. Envisioning the influence of social interactions, the objective of the current study is to investigate: (1) how should firms make operational decisions in terms of capacity and price; (2) what will be the equilibrium capacity and price decisions in service competition; (3) can firms achieve more profit from social interactions in competition; (4) can consumers benefit from social interactions with more utility or surplus from purchase decision-making. To answer these questions, we consider operations of two competing firms which provide substitutable products or services in capacity and price decisions in a market of a fixed size. For ease of exposition, we assume substitutable services are offered by profit maximizing firms. Service is a broad term in this chapter, which may refer to actual services offered by service providers, or the products supplied by make-to-order firms, etc. The firm is operated as a queueing system, and the capacity is modeled as the service rate. The market size is captured by the total potential arrival rate. Firms can expand the market by serving more consumers through capacity investment and price reduction. Capacities are costly which constrain the operational strategy in both capacity and price decision-making.

In the consumer market, social interactions influence consumer purchase decisions. Specifically, consumers may perceive the value of a service more if a greater

[4] http://www.nytimes.com/2010/06/09/dining/09reservations.html?pagewanted=all

[5] http://www.asianewsnet.net/news-34738.html

number of other customers make the same choice. Our results indicate that social interactions can always benefit a monopolist firm, where a larger market can be covered and a higher price can be charged, while a smaller capacity can be adopted. Under competition with two homogeneous firms, we focus on the symmetric equilibrium service rate and price decisions. Based on the equilibrium operational decisions, our result indicates that, social interactions and competition as well as their interplay may drive firms to build a high capacity, charge a low price, leading to a small profit, especially in a small market with more intense social interactions. However, if social interactions are less intense, or the market size is large, firms can still benefit from social interactions by charging a high price and building a small capacity under competition.

The remaining chapter is organized as follows. In Section 2.3, we briefly review several related studies about competition in service systems. Section 2.4 investigates the optimal service rate and price decisions for a monopolist. Consumer queue-joining behavior facing two competing firms is discussed in Section 2.5. We focus on service rate competition in Section 2.6. Price competition is studied in Section 2.7. In Section 2.8, we investigate the competition on service rate and price simultaneously under a fixed profit margin policy. Section 2.9 concludes this chapter with summary and managerial insights.

2.3 Related Studies

There are two streams of literature related to this study, namely the influence of social interactions in consumer purchase decisions, and the competition in service systems based on queueing framework. For excellent reviews about the research progress with the terminologies as well as the modeling issues on social interactions from various disciplines, see Manski (2000) and Hartmann et al. (2008). The influence of social interactions studied in this chapter includes the reference group effect, observational learning, word-of-mouth communication or network externality, etc. As the first stream has been generally reviewed in Chapter 1, we mainly focus on the second stream in this section.

Capacity and price decisions in a service system in operations research have been studied for several decades (Stidham, 2002; Ata and Shneorson, 2006). The model developed in this chapter is also related to the equilibrium analysis of queueing systems with strategic consumers as in Hassin and Haviv (2003), pioneered by Naor (1969), and the competition in queueing systems. Due to strategic behaviors, consumers face the choices of whether or not to join a queue, or which of several queues to join. Bell and Stidham (1983) study consumer queue-joining decisions with unobservable queue length in a fixed consumer market. They compare the optimal arrival rate allocations under the social optimum situation and the case with self-interest consumers. Hassin and Haviv (2003) provide an excellent literature review in strategic consumer queue-joining behavior. Several studies investigate the capacity decision of a supplier by taking the equilibrium queue-joining decision of consumers who are not strategic, such as the allocation of jobs by one buyer to minimize the expected waiting time in Cachon and Zhang (2006, 2007).

Plenty of studies focus on the optimal service rate and price decisions in queueing systems. On the competition in service rate, Kalai et al. (1992) study a model with two competing exponential servers and Poisson arrivals assuming an increasing and convex cost of service rate per time unit. In the model, competition in service rate is considered as a means for capturing a larger market share in order to maximize long-run expected profit per time unit. Deneckere and Peck (1995) incorporate the competition among consumers into a two-stage game where several firms simultaneously determine prices and capacities under a stochastic demand setting. In the model, consumers randomly choose a firm to visit and the probability of being served depends on the capacity and other consumer adoption strategies. It is shown that there exists at most one pure strategy equilibrium. So (2000) studies the problem where many service firms compete for market share on price and service time to maximize the profit using delivery time guarantees. The results indicate that when firms are identical, the equilibrium price and time guarantee decisions are similar to the optimal solution as a monopolist. However, heterogeneous firms will exploit their distinctive firm characteristics to differentiate their services, such as the high capacity firms provide better time guarantees, while firms with lower operating costs offer lower prices. The differentiation becomes more acute as demands become more time-sensitive.

Queue length may impact consumer queue-joining decisions. Christ and Avi-Itzhak (2002) study a model with two exponential servers competing for Poisson arrivals on service rate, where potential consumers make joining decisions based on the queue length. The consumer utility in our settings is based on the *full price* model, which is also used in Chen and Wan (2003; 2005). They consider the price and capacity competition of two make-to-order firms modeled as two M/M/1 queues in a fixed size consumer market. Their results demonstrate that there may exist no equilibrium, a unique equilibrium, or multiple equilibria in terms of price or service rate decisions. However, the influence of social interactions in consumer purchase decisions is not considered in their model. Allon and Federgruen (2007) analyze a general market with competing service facilities on price and waiting time standard. In the model, service facilities can select their choice on price and waiting time standard simultaneously or one after the other. Each facility is modeled as an M/M/1 queue system. Allon and Federgruen (2008) generalize the model into

Tab. 2.1: Key variables and notations.

Variables	Notations	Variables	Notations
Total potential arrival rate	Λ	Intrinsic service value	v
Service price	p	Social value	$V^s(\lambda)$
Capacity/Service rate	μ	Waiting cost per unit time	w
Social interaction intensity	α	Parameter of service cost	β

G/GI/s systems, and consider the firms compete on price, waiting time standard, and simultaneous price and waiting time standard. They characterize how the capacity cost and decision sequence impacts the equilibrium decisions.

From the above literature, we can see previous studies mainly consider the capacity and price decisions from the supplier's perspective. Although some studies take the congestion effect into consumer queue-joining decisions, few studies have considered the impact of social interactions explicitly on operational decisions. There is a lack of studies considering the positive externality from social interactions and negative externality from congestion simultaneously. By focusing on operational decisions both in a monopoly setting and duopoly competition from firm perspective, our study fills the gap in literature on how operational decisions would be impacted under both positive and negative effect of social interactions.

2.4 Monopoly Capacity and Price Decisions Under Social Interactions

A profit maximizing monopolist firm offers a service to the consumer market of a fixed size Λ, where consumer purchase decisions are influenced by others' choices through social interactions. Specifically, due to social interactions, the *perceived value* of the service to a potential consumer depends on the number of the consumers who are making the same choice. The firm is operated through a queueing system, where the capacity measured by the service rate μ and the price p need to be decided. Capacity is costly for the firm to expand in order to provide a fast service. A queue always exists due to random arrivals of the demand and random service times. In many service offerings and product deliveries, consumers may not always observe the queues or waiting lines before making purchase decisions. Therefore, we assume the queue is unobservable, which is a typical assumption used in the literature. Consumers need to decide whether to join the queue to purchase the service. We assume once the purchase decision is made, consumers will join the queue without reneging. Therefore, in the following section, purchase decision and queue-joining decision are used interchangeably. Key notations used in this chapter are listed in Table 2.1.

2.4.1 Consumer Purchase Decision

Due to the influence of social interactions, from consumer perspective, the *perceived value* V of the service depends on the *intrinsic value* v of the service and the *social value* $v^s(\lambda)$, which depends on the effective arrival rate to the firm. For tractability, we assume an additive form of the perceived value as $V = v + v^s(\lambda)$ and a linear form of the social value $v^s(\lambda) = \alpha(\lambda - \lambda_b)$, where $\lambda_b \geq 0$ represents the base demand. Our model of social interactions is based on the definition given by Brock and Durlauf (2001), where social interactions refer to *"the utility or payoff an individual receives from a given action depends directly on the choices of others in that individual's reference group."* In this chapter, we assume $\lambda_b = 0$ without loss of generality. The parameter α is defined as the *social interaction intensity*, which may depend on firm characteristics or customer purchase behaviors. In this chapter, we assume α is exogenous given. If $\alpha > 0$, the influence of social interactions is positive, i.e., more arrivals will increase consumer perceived value of the service, maybe due to the bandwagon effect, positive consumption externality or the network externality nature of the service, etc; if $\alpha < 0$, the influence of social interactions is negative, i.e., more arrivals will decrease consumer perceived value of the service, due to the snob effect or negative consumption externality, etc; if $\alpha = 0$, we have the traditional case where social interactions do not play a role in consumer purchase decisions. In this chapter, we mainly focus on the positive influence of social interactions under $\alpha > 0$ on consumer purchase behavior. The result can be easily generalized to the situation with a negative social interaction intensity.

We assume the firm is operated as an M/M/1 queue for tractability (the model can be extended to more general queueing settings). Therefore, demand for the service arrives according to a Poisson process and the service time is exponentially distributed. Although the M/M/1 model may not capture all aspects of the system behavior, it captures many of the congestion-related phenomenon (Ata and Shneorson, 2006). Moreover, using this model, we are able to explicitly characterize the equilibrium decisions under social interactions. Therefore, the expected waiting cost given the service rate μ and the actual arrival rate λ (in queue and service) is

$$CW(\lambda, \mu) = \begin{cases} \frac{w}{\mu - \lambda}, & 0 \leq \lambda < \mu \\ +\infty, & \mu \leq \lambda \leq \Lambda \end{cases}.$$

Consumer purchase decision depends on the expected surplus and reservation surplus. Specifically, if the expected surplus is no less than the reservation value, consumers will join the queue to procure the service; otherwise, they will balk. Since the queue is unobservable, based on the full price model, consumer expected surplus is defined as

$$S(\lambda^E, \mu, p) = v + \alpha\lambda^E - p - CW(\lambda^E, \mu)$$

where λ^E is the anticipated arrival rate.

We assume consumers are *homogeneous* in terms of waiting cost per unit time, and normalize their reservation surplus to be 0 (the model can be generalized to the situation with heterogeneous customers with different waiting cost per unit time). Since the queue is unobservable, upon arrival, each consumer makes the joining decision depending on the perceived value of the service, the expected waiting cost based on their anticipated arrival rate, and the price, before the actual arrival rate is known. We assume consumers are *strategic* in queue-joining decisions in the sense that, if the expected surplus is positive, they will join the queue; if the expected surplus is negative, they will not join the queue; in equilibrium, the anticipated arrival rate and the effective arrival rate are coincident $\lambda^E = \lambda$, as in the *fulfilled expectation* (or rational expectation) framework. For individual consumers, we focus on the *symmetric equilibrium* queue-joining strategy as in Hassin and Haviv (2003) and Anand et al. (2011). Specifically, letting $\gamma^e(\mu, p)$ denote the equilibrium probability that an individual consumer would join the queue given the service rate μ and price p, we have the following scenarios:

- If $S(\Lambda, \mu, p) = v + \alpha\Lambda - CW(\Lambda, \mu) - p > 0$, i.e., the expected surplus is nonnegative even if a potential customer anticipates all the other consumers join the queue; therefore, all consumers will join the queue in equilibrium, i.e., $\gamma^e(\mu, p) = 1$;
- If $\max_{\lambda\in[0,\Lambda]} S(\lambda, \mu, p) < 0$, i.e., the expected surplus is negative for any $\lambda \in [0, \Lambda]$; therefore, no consumers will join the queue in equilibrium, i.e., $\gamma^e(\mu, p) = 0$;
- If $\exists\lambda \in [0, \Lambda],\ S(\lambda, \mu, p) = v + \alpha\lambda - CW(\lambda, \mu) - p = 0$, i.e., each consumer plays a mixed strategy in equilibrium, in the sense that each consumer will join the queue with probability $\gamma^e(\mu, p) = \frac{\lambda}{\Lambda} \in [0, 1]$.

In the third case, there may exist two points $\lambda_{(1)}$ and $\lambda_{(2)}$ in the range $[0, \Lambda]$, such that $S(\lambda, \mu, p) = 0$, since the expected surplus function is concave in λ. We always refer to the larger one as the equilibrium arrival rate in this chapter. Therefore, for the monopolist, given the service rate μ and price p, the effective arrival rate $\lambda(\mu, p)$ is given as the following result:

$$\lambda(\mu, p) = \begin{cases} \Lambda, & S(\Lambda, \mu, p) > 0 \\ \lambda \in [0, \Lambda], & S(\lambda, \mu, p) = 0 \\ 0, & \max_{\lambda\in[0,\Lambda]} S(\lambda, \mu, p) < 0 \end{cases}.$$

In the following section, we use the superscript m to denote the optimal service rate and price decisions for the monopolist. Although the above consumer purchase behavior is developed based on unobservable queues, the model can be also interpreted as the equilibrium result with observable queues.

2.4.2 Monopoly Optimal Price and Service Rate

Since consumer expected surplus is increasing in μ and decreasing in p, the monopolist can increase the service rate μ or lower the price p to serve more consumers per unit time. However, service rate is not free to increase. Following existing studies, the service rate cost is assumed to be $C(\mu) = \beta\mu$ which captures the marginal cost for each served customer and β measures the capacity sensitivity in service cost. Therefore, the expected profit of the monopolist with the service rate μ and price p is given as $\pi(\mu, p) = (p - \beta\mu)\lambda$ if the arrival rate is λ. Under social interactions, the objective of the monopolist is to maximize the expected profit through the following profit maximizing problem

$$\begin{aligned} \max_{\mu,p} \pi(\mu, p) \quad &= \quad \max_{\mu,p}(p - \beta\mu)\lambda \\ s.t. \quad & \quad v + \alpha\lambda - p - CW(\lambda, \mu) \geq 0 \\ & \quad \lambda \in [0, \Lambda],\ 0 \leq \lambda < \mu \end{aligned} \tag{2.4.1}$$

We assume the following condition holds in this chapter, $v \geq 2\sqrt{\beta w}$. This assumption indicates that even if there are no social interactions, consumers will join the queue if the price is charged as low as the service rate cost ($p = \beta\mu$), i.e., $v \geq 2\sqrt{\beta w} \rightarrow v - \beta\mu - \frac{w}{\mu} \geq 0$ if $\mu \in [\frac{v-\sqrt{v^2-4\beta w}}{2\beta}, \frac{v+\sqrt{v^2-4\beta w}}{2\beta}]$, which indicates the monopolist can always achieve positive profit under appropriate operational decisions, since the operational space for the service rate is non-empty.

The optimal decisions for the monopolist are given as the following result:

Proposition 2.1. *The optimal service rate and price are* $\mu^m = \lambda^m + \sqrt{\frac{w}{\beta}}$, *and* $p^m = v + \alpha\lambda^m - \sqrt{\beta w}$, *with the profit* $\pi^m = (\alpha - \beta)\lambda^{m2} + (v - 2\sqrt{\beta w})\lambda^m$, *where*

$$\lambda^m = \begin{cases} \min\left(\frac{v-2\sqrt{\beta w}}{2(\beta-\alpha)}, \Lambda\right), & \alpha < \beta \\ \Lambda, & \alpha \geq \beta \end{cases}$$

All omitted proofs in this chapter are listed in Appendix 6.1. Therefore, social interactions play an important role in operations. From the above result we can see, if social interactions are intense, such that $\alpha \geq \beta$, it is always optimal for the monopolist to cover the whole market. The reason can be explained by the following pricing policy $p = \beta\mu + \Delta$, where for each consumer, the facility charges a fixed premium (or the profit margin) $\Delta \geq 0$ and $\Delta \leq v + (\alpha - \beta)\Lambda - 2\sqrt{\beta w}$. Suppose we increase the service rate by $\delta \geq 0$ at the cost of $\beta\delta$, if the arrival rate also increases by δ, the expected waiting cost keeps the same. Since the social value will increase by $\alpha\delta$, the price can be increased by $\alpha\delta$ to keep consumer expected surplus as zero. However, the increased price is larger than the increased service rate cost, i.e., $\alpha\delta \geq \beta\delta$, which is beneficial to the monopolist. Therefore, the monopolist will always cover the whole market for any fixed premium. However, if $\alpha < \beta$, it is not always optimal to cover the whole market. Whether it is optimal to cover the whole market depends on the service value, the service rate cost, and the waiting cost per unit time. While if α is approaching β, it may be always optimal for the monopolist to cover the whole market. The above result is also applicable to the case $\alpha < 0$, the negative influence of social interactions. We can see when $\alpha < 0$, the monopolist has to charge a lower price and covers a smaller market, compared with the situation without social interactions where $\alpha = 0$. The above result also implies, under the optimal monopoly decisions, from consumer perspective, the average waiting time is constant as $\sqrt{\frac{\beta}{w}}$ which is independent of the social interaction intensity, while the average queue length $\lambda^m\sqrt{\frac{\beta}{w}}$ depends on social interactions if the market is not fully covered, i.e., the average queue length is increasing in the social interaction intensity.

2.4.3 Managerial Implication

Compared with the result where social interactions are absent, i.e., $\alpha = 0$, if the market is fully covered, the service rates will be the same; however, the price charged under social interactions can be higher, which brings a larger profit. The reason is due to social interactions, consumer perceived value of the service becomes larger if more consumers are adopting the same service, which drive them to pay more. Under the same market coverage, although the service rates are the same under $\alpha > 0$ and $\alpha = 0$, the price is always higher under $\alpha > 0$. Without social interactions, the monopolist may only partially cover the market, while under social interactions, the monopolist can always cover a larger market or even the whole market.

The operational decisions under $\alpha = 0$ also capture the situation where managers ignore the influence of social interactions. Therefore, if managers ignore social interactions when making operational decisions, a potential profit will be lost due to a lower price or a smaller market coverage. In the following section, we compare the potential profit loss when managers ignore social interactions. We assume the potential market size Λ is large enough, such that the market will be partially covered without social interactions. Suppose the monopolist sets the price and the service rate without considering social interactions, i.e., $p^o = v - \sqrt{\beta w}$, $\mu^o = \frac{v}{2\beta}$ and

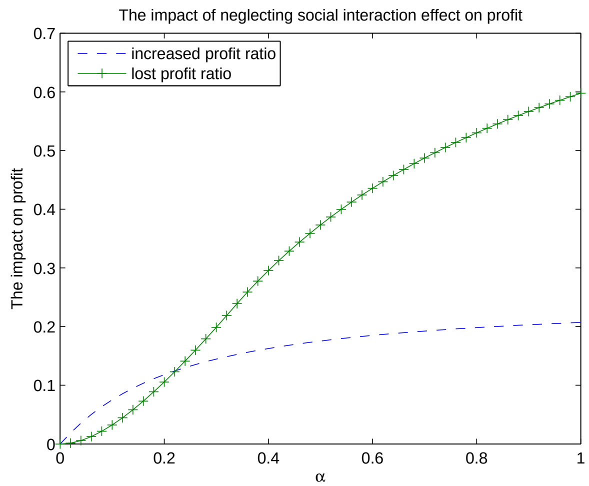

Fig. 2.4.1: The impact of ignoring social interactions on the profit with respect to the social interaction intensity α.

$\lambda^o = \frac{v-2\sqrt{\beta w}}{2\beta}$. The actual arrival rate denoted as λ^a under social interactions can be solved from the following equation:

$$v + \alpha\lambda - \frac{w}{\mu^o - \lambda} - p^o = 0 \Rightarrow \lambda = \frac{\alpha v - 2\beta\sqrt{\beta w} + \sqrt{(2\beta\sqrt{\beta w} + \alpha v)^2 - 16\alpha\beta^2 w}}{4\alpha\beta},$$

and the actual arrival rate is $\lambda^a = \min(\lambda, \Lambda)$.

Based on the assumption $v > 2\sqrt{\beta w}$, one of the solutions of λ is always negative, while the other one is in $(0, \frac{v}{2\beta})$. Therefore, the effective arrival rate λ^a is always larger than the result without social interactions, i.e., $\lambda^o < \lambda^a$. Define $\pi^N(\mu^o, p^o)$ as the optimal profit without social interactions, $\pi(\mu^o, p^o, \lambda^a)$ as the actual profit under social interactions, $\pi^Y(\mu^m, p^m)$ as the optimal profit with social interactions. Therefore, $\frac{\pi(\mu^o,p^o,\lambda^a)-\pi^N(\mu^o,p^o)}{\pi^N(\mu^o,p^o)}$ is the *increased profit ratio* due to social interactions compared with the case without social interactions, and $\frac{\pi^Y(\mu^m,p^m)-\pi(\mu^o,p^o,\lambda^a)}{\pi^Y(\mu^m,p^m)}$ is the *lost profit ratio* due to ignoring social interactions. Fig.2.4.1 describes the increased profit ratio and the lost profit ratio under the parameters $v = 10,\ \beta = 1,\ w = 1,\ \Lambda = 6$, where the optimal price and service rate without social interactions are $p^o = 9,\ \mu^o = 5$ and $\lambda^o = 4$.

From the figure, we can see, the influence of social interactions in consumer purchase decisions always benefits the monopolist, even if operational decisions are made without considering this influence. The increased profit ratio is always increasing in social interaction intensity. However, compared with the case where social interactions are considered, the lost profit ratio is also increasing in social interaction intensity but more significant if social interaction intensity is large. Therefore, we can see firms will lose substantial profit if ignoring social interactions in operational decision-making. While considering the influence in consumer purchase decision, the monopolist will obtain more profit.

2.5 Service Competition Under Social Interactions

From the above section, we can see for a monopolist, the influence of social interactions in consumer purchase decisions always benefits the firm, where a larger market can be covered and a higher price can be charged. If there are many competing suppliers which offer similar or substitutable services, consumers have many alternatives when making purchase decisions. Whether social interactions can benefit firms under competition needs further investigation. We focus on the competition between two firms which provide perfectly substitutable services in the market with a fixed potential size Λ. The models can be generalized to the case with multiple competing firms.

The two firms may be *symmetric* (or homogeneous) if the service rate cost and social interaction intensity in both firms are the same respectively, *asymmetric* (or heterogeneous) otherwise. They are competing for market shares through capacity and price decisions. We use the subscript i to denote the parameters and decisions for firm i. We focus on the equilibrium operational decisions, defined in the following section. The sequence of the events is as follows: the two firms simultaneously make the price and the service rate decisions; customers make queue-joining decisions after observing the price and service rate from each firm.

Since queues of both firms are unobservable before joining the system, arriving consumers need to decide which queue to join to purchase the service or balk without entering any queue. We also assume consumers are homogeneous with the same waiting cost per unit time as w from either firm. (It is possible that customers may be subject to different waiting costs from different firms.) Consumer queue-joining decisions depend on the expected surplus from either service provider. Denote the expected surplus from firm i as $S(\lambda_i^E, \mu_i, p_i) = v + \alpha\lambda_i^E - CW(\lambda_i^E, \mu_i) - p_i$, $i = 1, 2$, where an arriving consumer expects λ_i^E consumers would join the same queue. We consider all consumers adopt the same mixed queue-joining strategy denoted as $\gamma_i \geq 0, i = 0, 1, 2$, and $\sum \gamma_i = 1$, where γ_0 denotes the probability of balking; γ_1 denotes the probability of joining the queue of the first firm; γ_2 denotes the probability of joining the queue of the second firm. In the equilibrium of consumer queue-joining decisions, the expected arrival rate and the effective arrival rate are consistent as $\lambda_i^E = \lambda_i = \Lambda\gamma_i$.

We assume consumers always try to maximize the expected surplus when making queue-joining decisions. In terms of the market coverage, given the service rate μ_i and the price p_i of each firm, we have the following situations:

1. The market is fully covered, and each firm covers a strictly positive market, i.e., $\lambda_i > 0$, $\lambda_1 + \lambda_2 = \Lambda$, $i = 1, 2$;

2. The market is fully covered by one firm, i.e., $\lambda_i = 0$ and $\lambda_j = \Lambda$, $i \neq j$;

3. The market is partially covered, i.e., $\lambda_i \geq 0$, $\lambda_1 + \lambda_2 < \Lambda$, $i = 1, 2$.

For the first case, consumer expected surplus from either firm satisfies $S(\lambda_1, \mu_1, p_1) = S(\lambda_2, \mu_2, p_2) \geq \max(S(\Lambda, \mu_1, p_1), S(\Lambda, \mu_2, p_2), 0)$. For the second case, the expected surplus from joining the queue of firm j is the largest among all possible mixed queue-joining decisions. For the third case, the expected surplus from either firm is the same as the reservation value, i.e., $S(\lambda_1, \mu_1, p_1) = S(\lambda_2, \mu_2, p_2) = 0$.

The effective arrival rate to each firm depends on the service rate and the price decisions, denoted as $\lambda_i = G(\mu_i, \mu_j, p_i, p_j)$. The profit function of firm i with service rate μ_i and price p_i, given the other firm chooses μ_j and p_j is denoted as

$$\pi(\mu_i, p_i, \mu_j, p_j) = (p_i - \beta\mu_i)\lambda_i = (p_i - \beta\mu_i)G(\mu_i, \mu_j, p_i, p_j)$$

and the objective of firm i is to maximize the expected profit. We consider the optimal service rate and price of each firm in the *Nash equilibrium* under service competition, denoted as μ_i^e and p_i^e respectively. In Nash equilibrium, consumer queue-joining strategy is the best response of firms' operational decisions, i.e., given the service rate and the price of each firm, consumers try to maximize their expected surplus through queue-joining decisions; for each firm, the operational decisions (service rate, price or both) are the best response of the competitor's decisions and the consumer queue-joining decisions.

Specifically, the following defines the *Nash equilibrium* in operations: given the other firm j chooses $\mu_j = \mu_j^e$ and $p_j = p_i^e$, the profit for firm i with operational decisions $\mu_i \neq \mu_i^e$ and $p_i \neq p_i^e$ is always suboptimal, as

$$\forall \mu_i \neq \mu_i^e,\ p_i \neq p_i^e,\ \pi(\mu_i^e, p_i^e, \mu_j^e, p_j^e) \geq \pi(\mu_i, p_i, \mu_j^e, p_j^e).$$

Based on the monopoly optimal decisions, we can immediately get the following result in terms of the equilibrium market coverage in service competition:

Lemma 2.1. *The following result in terms of the market coverage holds:*

- *If at least one firm has a large social interaction intensity, such that $\alpha_i \geq \beta_i$, the market will be fully covered for any potential market size Λ and consumer expected surplus is non-negative.*

- *If the social interaction intensity for each firms is small, such that* $\alpha_i < \beta_i$, $i = 1, 2$,
  - *under* $\Lambda \leq \frac{v-2\sqrt{\beta_1 w}}{2(\beta_1-\alpha_1)} + \frac{v-2\sqrt{\beta_2 w}}{2(\beta_2-\alpha_2)}$, *the market will be fully covered and consumer expected surplus is non-negative;*
  - *under* $\Lambda > \frac{v-2\sqrt{\beta_1 w}}{2(\beta_1-\alpha_1)} + \frac{v-2\sqrt{\beta_2 w}}{2(\beta_2-\alpha_2)}$, *the market is partially covered, and each firm operates as a monopolist, with the operational decisions* $\mu_i^m = \lambda_i^m + \sqrt{\frac{w}{\beta_i}}$, $p_i^m = v+\alpha_i\lambda_i^m - \sqrt{\beta_i w}$, *and covers* $\lambda_i = \frac{v-2\sqrt{\beta_i w}}{2(\beta_i-\alpha_i)}$; *consumer expected surplus is 0 from either firm.*

The proof of the above result is straightforward, which is omitted here. Therefore, if the social interaction intensity is large enough at one of the two firms or both firms, the market will be fully covered. The result is intuitive. Suppose the market is not fully covered in equilibrium as $\lambda_1 + \lambda_2 < \Lambda$. Then consumer expected surplus must be zero from either firm. However, since $\alpha_i \geq \beta_i$, the optimal decision for firm i is to cover the market as large as possible based on the monopoly result. Therefore, the partial market coverage can not be an equilibrium. If the social interaction intensity is small at both firms, the market coverage depends on the potential market size Λ. If the potential market size is smaller than the total monopoly optimal market sizes of the two firms, the market will also be fully covered; while if the potential market size is large enough, the market will be partially covered.

Given each firm's price and service rate decisions, when the social interaction intensity is large enough, all customers may join the same queue and leave the other queue empty. The following result provides a possible situation for one firm to cover the whole market:

Lemma 2.2. *If* $\mu_i > \Lambda$ *and* $\alpha_i \geq \frac{w}{(\mu_i-\Lambda)^2}$, *then the market will be fully covered by one firm. Specifically, if* $v + \alpha_i\Lambda - \frac{w}{\mu_i-\Lambda} - p_i \geq \max(S(\Lambda, \mu_j, p_j), 0)$, *the market is fully covered by firm* i*; while if* $S(\Lambda, \mu_j, p_j) \geq \max(v + \alpha_i\Lambda - \frac{w}{\mu_i-\Lambda} - p_i, 0)$, *then the market is fully covered by firm* j.

Therefore, under social interactions, one firm may cover the whole market while leaving its competitor with empty demand, especially when the social interactions are intense. As discussed in Chen and Wan (2003; 2005), there may exist no equilibrium or multiple equilibria in price competition and service rate competition for two symmetric firms based on the full price model even if $\alpha = 0$. In the following sections, we also focus on the competition between two symmetric firms. Since there are two operational decisions, there may exist multiple combinations of the service rate and price decisions in equilibrium. To reduce analytical complexity, in the following section, we mainly focus on the competition in one-dimensional decisions. Specifically, we will investigate the service rate competition with fixed price constraint, the price competition with fixed service rate constraint, and the simultaneous service rate and price competition under fixed *profit margin policy*. In each scenario, we first consider the optimal monopoly decision, and then investigate the equilibrium operational decision under competition. As discussed in Brock and Durlauf (2001), when social interactions act as strategic complementarity among agents, multiple equilibria may occur in absence of any coordination mechanisms. Therefore, in the following sections, appropriate assumptions will be stated if necessary in each scenario to exclude multiple equilibria and symmetric equilibrium is primarily investigated.

2.6 Service Rate Competition Under Social Interactions

In this section, we consider the two firms compete on the service rate where the price is assumed to be fixed. The service speed can be considered as one aspect of the service quality. The fixed service price may be due to regulations. The two firms may be operated by the same company where service rate will be individually determined as the situation in Anand et al. (2011).

2.6.1 Optimal Monopoly Service Rate

Given the fixed price p, the monopoly profit optimization problem is formulated as

$$\begin{aligned} \max \quad & (p-\beta\mu)\lambda \\ s.t. \quad & v+\alpha\lambda-p-CW(\lambda,\mu)\geq 0, \lambda\in[0,\Lambda] \end{aligned} \tag{2.6.1}$$

For the monopolist, in equilibrium, consumer expected surplus must be zero under the optimal service rate decision. Therefore, given the arrival rate, the service rate satisfies

$$\forall\lambda\in[0,\Lambda],\ \mu=\lambda+\frac{w}{v-p+\alpha\lambda}$$

which indicates if $\frac{\alpha w}{(v-p+\alpha\Lambda)^2}\geq 1$, μ is decreasing in $\lambda\in[0,\Lambda]$; while if $\frac{\alpha w}{(v-p)^2}\leq 1$, μ is increasing in $\lambda\in[0,\Lambda]$. It is possible that if $\frac{\alpha w}{(v-p+\alpha\Lambda)^2}\leq 1$ and $\frac{\alpha w}{(v-p)^2}\geq 1$, μ is first decreasing in $\lambda\in[0,\lambda^c]$, and then increasing in $(\lambda^c,\Lambda]$, where $\frac{\alpha w}{(v-p+\alpha\lambda^c)^2}=1$. In this section, we assume $\frac{\alpha w}{(v-p)^2}\leq 1$, so that μ is always increasing in $\lambda\in[0,\Lambda]$, i.e., given μ, there always exists a unique $\lambda<\mu$. We also assume $p-\frac{\beta w}{v-p}\geq 0$, so that a positive market size will be covered under the optimal service rate decision, since if $\lambda=0$, then $\mu=\frac{w}{v-p}$, the profit margin is always non-negative as $p-\beta\mu\geq 0$.

The monopoly profit optimization problem is reduced as

$$\max_{\lambda\in[0,\Lambda]} \pi(\lambda,p) = \max_{\lambda\in[0,\Lambda]} \left(p - \beta\lambda - \frac{\beta w}{v-p+\alpha\lambda}\right)\lambda$$

with the first and second order derivatives with respect to λ as

$$\frac{\partial\pi(\lambda,p)}{\partial\lambda} = p - 2\beta\lambda - \frac{\beta w(v-p)}{(v+\alpha\lambda-p)^2}, \quad \frac{\partial^2\pi(\lambda,p)}{\partial\lambda^2} = -2\beta\left[1 - \frac{\alpha w(v-p)}{(v+\alpha\lambda-p)^3}\right]$$

where we can see the second order derivative is decreasing in λ.

Based on the assumption $\frac{\alpha w}{(v-p)^2} \leq 1$, $\forall\lambda \in [0,\Lambda]$, $\frac{\partial^2\pi(\lambda,p)}{\partial\lambda^2} \leq 0$ indicates $\pi(\lambda,p)$ is concave in λ. Therefore, we have the following result in terms of the optimal monopoly service rate with the fixed price constraint:

Lemma 2.3. *Given the fixed price p, the optimal service rate is $\mu^m = \lambda^m + \frac{w}{v-p+\alpha\lambda^m}$, where λ^m is the optimal arrival rate, as $\lambda^m = \min(\lambda^\star, \Lambda)$, where $\lambda^\star$ satisfies the first order condition*

$$\frac{\partial\pi(\lambda,p)}{\partial\lambda} = p - 2\beta\lambda^\star - \frac{\beta w(v-p)}{(v+\alpha\lambda^\star-p)^2} = 0.$$

Therefore, given the market size Λ and the fixed price p, we can see the social interaction intensity α plays an important role in the service rate decision for the monopolist. The firm can cover the whole market or partially cover the market depending on the social interaction intensity α, the waiting cost per unit time w, the net value of the service $v-p$, and the total potential market size Λ, as well as the marginal cost of service rate β. For example, given all the other parameters fixed, if α increases, $\lambda^\star$ also increases, since

$$\frac{\partial\lambda^\star}{\partial\alpha} = \frac{2\beta w(v-p)\lambda^\star}{2\beta\left[(v+\alpha\lambda^\star-p)^3 - \alpha w(v-p)\right]} > 0$$

Therefore, when α is large enough, $\lambda^\star \geq \Lambda$, the monopolist may cover the whole market. Specifically, if $\Lambda \leq \frac{p}{2\beta} - \frac{w}{2(v-p)}$, the monopolist will cover the whole market for any $\alpha \geq 0$.

Given a fixed total market size Λ, Fig.2.6.1-2.6.2 illustrate the impact of price level given fixed social interaction intensity, and the impact of social interaction intensity given fixed price level on the optimal service rate decision in the monopoly setting. The following parameters are used in the numerical example, $v = 10$, $w = 1$, $\beta = 1$, $\Lambda = 4$.

Given the fixed social interaction intensity, when the price level increases, both service rate and arrival rate increase. While given the fixed price level, if the social

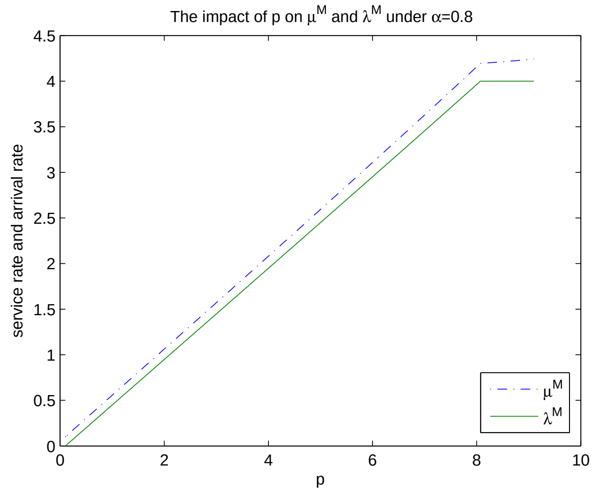

Fig. 2.6.1: The impact of the price p on the optimal service rate and arrival rate under the social interaction intensity $\alpha = 0.8$.

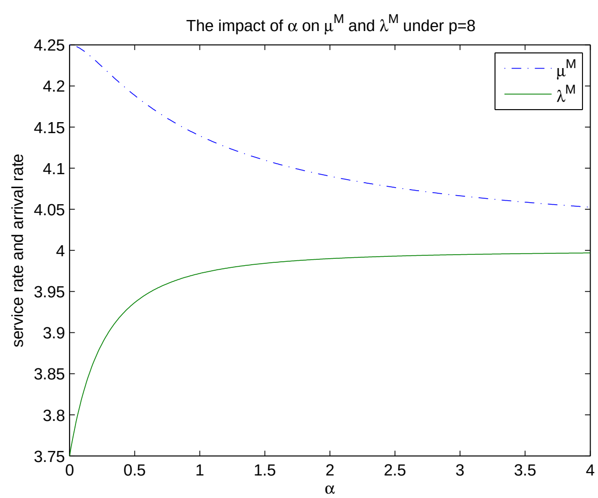

Fig. 2.6.2: The impact of the social interaction intensity α on the optimal service rate and arrival rate under the price $p = 8$.

interaction intensity increases, the arrival rate will increase, while the service rate will decrease. Therefore, the monopolist will get more profit if social interaction intensity become large, since a smaller service rate will be determined, while a larger market will be covered.

2.6.2 Duopoly Service Rate Competition

Based on the monopoly result, we can see for the fixed price level p, if the monopoly optimal arrival rate is small, such that $\lambda^m \leq \frac{\Lambda}{2}$, service competition does not impact service rate decisions, since each firm will be operated as a monopolist. The condition for the situation where competition does not impact service rate decisions is

$$p - \beta\Lambda - \frac{\beta w(v-p)}{(v+\alpha\frac{\Lambda}{2}-p)^2} \leq 0$$

where the left hand side (LHS) is decreasing in Λ based on the assumption $\frac{\alpha w}{(v-p)^2} \leq 1$. Therefore, if the total potential market size is large enough, such that $\Lambda \geq \Lambda^c$, where Λ^c satisfies the above equation, each firm will be operated as a monopolist. The conclusion can be easily generalized to the competition with $n \geq 2$ symmetric firms, where if $\Lambda \geq \frac{n\Lambda^c}{2}$, each firm will be operated as a monopolist.

However, if $\Lambda < \Lambda^c$, as a monopolist, each firm wants to cover a larger market than $\frac{\Lambda}{2}$ or even the whole market. Therefore, competition impacts their service rate decisions, which is the focus of the following section.

We first have the result that the market will be fully covered in equilibrium:

Lemma 2.4. *Under the above condition, the market will be fully covered in equilibrium, i.e.,* $\lambda_1 + \lambda_2 = \Lambda$.

Proof. Suppose under the above condition, the market is not fully covered in equilibrium, i.e., at least one of the firms covers a small market which is less than $\frac{\Lambda}{2}$. Thus, there exists a portion of customers balking from either queue, which indicates customer expected surplus must be 0. Therefore, the firm whose market share is less than $\frac{\Lambda}{2}$ can always increase its profit by covering an additional market which increases its profit. Therefore, in equilibrium, the market must be fully covered. □

Based on the above result, there are three possible results for the full market coverage situation, namely one firm covers the whole market and leaves the other firm empty, i.e., $\lambda_i = \Lambda$ and $\lambda_j = 0$; or the two firms split the whole market with strictly positive market share, i.e., $0 < \lambda_i < \Lambda$ and $\lambda_i + \lambda_j = \Lambda$. Specifically, if the total market is small enough, such that $p - 2\beta\Lambda - \frac{\beta w(v-p)}{(v+\alpha\Lambda-p)^2} \geq 0$, then one firm may cover the whole market; while if $p - 2\beta\Lambda - \frac{\beta w(v-p)}{(v+\alpha\Lambda-p)^2} < 0$, then the two firms split the market and each one will cover a strictly positive market.

Similar to the monopoly setting, the service rate is bounded as $\mu \in [\frac{w}{v-p}, \frac{p}{\beta}]$ to insure a non-negative arrival rate and profit. Since we consider the competition between two symmetric firms, we mainly focus on the symmetric equilibrium where both firms make the same service rate decision and cover the same size of market. In the symmetric equilibrium, to exclude the case where one firm covers the whole market, we assume $\forall \mu \in [\frac{w}{v-p}, \frac{p}{\beta}]$, and $\mu > \Lambda$, $S(\Lambda, \mu, p) \leq S(\frac{\Lambda}{2}, \mu, p)$, i.e., $\alpha(\frac{p}{\beta} - \Lambda)(\frac{p}{\beta} - \frac{\Lambda}{2}) \leq w$, which indicates, even if both firms adopt the maximum service rate $\frac{p}{\beta}$, the expected surplus from joining the same queue is always smaller when one half of the consumers join each queue. Therefore, consumers will not join the same queue in any symmetric equilibrium for $\mu^e \in [\frac{w}{v-p}, \frac{p}{\beta}]$, and each firm will cover a strictly positive market. To exclude multiple equilibria, we assume consumers always join the queue of a larger service rate with a larger probability, i.e., if $\mu_1 > \mu_2$, then $\lambda_1 > \frac{\Lambda}{2} > \lambda_2$.

In the symmetric equilibrium, each firm sets the service rate at $\mu_i = \mu^e$ and covers $\lambda_i^e = \frac{\Lambda}{2}$. Clearly, $\mu^e \geq \frac{\Lambda}{2} + \frac{w}{v-p+\alpha\frac{\Lambda}{2}}$. We investigate the condition under which the symmetric equilibrium holds and whether $\mu^e = \frac{\Lambda}{2} + \frac{w}{v-p+\alpha\frac{\Lambda}{2}}$ can be a symmetric equilibrium in service competition under social interactions. It turns out that the total market size plays an important role in equilibrium service rate decision.

We have the following result in terms of the equilibrium service rate if the total market size is intermediate:

Proposition 2.2. *If* $\frac{3\beta\Lambda}{2} - p + \frac{\beta w(v-\alpha\frac{\Lambda}{2}-p)}{(v+\alpha\frac{\Lambda}{2}-p)^2} > 0$*, in the symmetric equilibrium, each firm covers one half of the market* $\frac{\Lambda}{2}$*, and the symmetric equilibrium service rate is* $\mu^e = \frac{\Lambda}{2} + \frac{w}{v-p+\alpha\frac{\Lambda}{2}}$ *. If the following conditions hold,*

$$\left[p - \beta\left(\frac{\Lambda}{2} + \frac{w}{v-p+\alpha\frac{\Lambda}{2}}\right)\right]\frac{(v-p+\alpha\frac{\Lambda}{2})^2}{2[(v-p+\alpha\frac{\Lambda}{2})^2 - \alpha w]} - \frac{\beta\Lambda}{2} \leq 0,$$

and

$$\forall \delta \in [0, \frac{p}{\beta} - \mu^e],\ (p - \beta\mu^e - \beta\delta)\epsilon''(\delta) - 2\beta\epsilon'(\delta) \leq 0$$

where ϵ *satisfies* $F(\delta, \epsilon) = 2\alpha\epsilon\left(\mu^e + \delta - \frac{\Lambda}{2} - \epsilon\right)\left(\mu^e - \frac{\Lambda}{2} + \epsilon\right) - 2w\epsilon + w\delta = 0$*,* $\epsilon'(\delta)$ *and* $\epsilon''(\delta)$ *are the first and second order conditions for* ϵ *in terms of* δ*, then the symmetric equilibrium service rate* $\mu^e = \frac{\Lambda}{2} + \frac{w}{v-p+\alpha\frac{\Lambda}{2}}$ *is the unique Nash equilibrium.*

Based on the assumption $\frac{\alpha w}{(v-p)^2} \leq 1$, it can be proved that the LHS of the above condition is increasing in Λ. Therefore, there exists a critical Λ^{c1}, such that if $\Lambda > \Lambda^{c1}$, the above condition always holds. At $\Lambda = \Lambda^c$, it can be proved that $\frac{3\beta\Lambda^c}{2} - p + \frac{\beta w(v-\alpha\frac{\Lambda^c}{2}-p)}{(v+\alpha\frac{\Lambda^c}{2}-p)^2} > 0$, since $\frac{3\beta\Lambda^c}{2} - p + \frac{\beta w(v-\alpha\frac{\Lambda^c}{2}-p)}{(v+\alpha\frac{\Lambda^c}{2}-p)^2} = -p + \beta\Lambda^c + \frac{\beta w(v-p)}{(v+\alpha\frac{\Lambda^c}{2}-p)^2} + \frac{\beta\Lambda^c}{2}(1 - \frac{\alpha w}{(v+\alpha\frac{\Lambda^c}{2}-p)^2}) > 0$. Therefore, $\Lambda^{c1} < \Lambda^c$, which indicates if the total potential market size Λ is in the range $(\Lambda^{c1}, \Lambda^c)$, there exists a symmetric equilibrium, where each firm sets the service rate at $\mu^e = \frac{\Lambda}{2} + \frac{w}{v-p+\alpha\frac{\Lambda}{2}}$ and covers one half of the market. In the symmetric equilibrium, consumer expected surplus from either firm is zero, i.e., consumers can not get any benefit from social interactions even if there are several competing firms providing similar services.

If the total market size is small, we have the following result in terms of the service rate decision in the symmetric equilibrium:

Proposition 2.3. *If $\frac{3\beta\Lambda}{2} - p + \frac{\beta w(v-\alpha\frac{\Lambda}{2}-p)}{(v+\alpha\frac{\Lambda}{2}-p)^2} \leq 0$, in the symmetric equilibrium, each firm covers one half of the market $\frac{\Lambda}{2}$, and the symmetric equilibrium service rate μ^e satisfies*

$$\mu^e = \frac{p}{\beta} - \Lambda + \alpha\Lambda\frac{\left(\mu^e - \frac{\Lambda}{2}\right)^2}{w}.$$

If the following condition holds,

$$\forall\delta \in [0, \frac{p}{\beta} - \mu^e],\ (p - \beta\mu^e - \beta\delta)\epsilon''(\delta) - 2\beta\epsilon'(\delta) \leq 0$$

where ϵ satisfies $F(\delta,\epsilon) = 2\alpha\epsilon\left(\mu^e + \delta - \frac{\Lambda}{2} - \epsilon\right)\left(\mu^e - \frac{\Lambda}{2} + \epsilon\right) - 2w\epsilon + w\delta = 0$, $\epsilon'(\delta)$ and $\epsilon''(\delta)$ are the first and second order conditions for ϵ in terms of δ, then the symmetric equilibrium is the unique Nash equilibrium.

In the above result, if $\mu^e = \frac{\Lambda}{2} + \frac{w}{v+\alpha\frac{\Lambda}{2}-p}$, we have the condition $\frac{3\beta\Lambda}{2} - p + \frac{\beta w(v-\alpha\frac{\Lambda}{2}-p)}{(v+\alpha\frac{\Lambda}{2}-p)^2} = 0$; if $\mu^e > \frac{\Lambda}{2} + \frac{w}{v+\alpha\frac{\Lambda}{2}-p}$, we have the condition $\frac{3\beta\Lambda}{2} - p + \frac{\beta w(v-\alpha\frac{\Lambda}{2}-p)}{(v+\alpha\frac{\Lambda}{2}-p)^2} < 0$. Therefore, if $\frac{3\beta\Lambda}{2} - p + \frac{\beta w(v-\alpha\frac{\Lambda}{2}-p)}{(v+\alpha\frac{\Lambda}{2}-p)^2} \leq 0$, i.e., $\Lambda \leq \Lambda^{c1}$, the symmetric equilibrium service rate satisfies $\mu^e \geq \frac{\Lambda}{2} + \frac{w}{v+\alpha\frac{\Lambda}{2}-p}$, which indicates consumers may get positive expected surplus when the total potential market size is small. Therefore, we have the following result:

Corollary 2.1. *If $\Lambda > \Lambda^{c1}$, consumer expected surplus in the symmetric equilibrium is 0; if $\Lambda \leq \Lambda^{c1}$, consumers can get positive expected surplus.*

The above result is not difficult to understand. If the total potential market size is small, each firm wants to cover the market as much as possible. Therefore, both firms will decide a higher service rate which leads to a strictly positive expected surplus for consumers. If the total potential market becomes large, to cover one half

Tab. 2.2: The impact of market size on the equilibrium service rate and consumer expected surplus under social interactions.

Market size	Equilibrium service rate	Consumer expected surplus
$\Lambda \leq \Lambda^{c1}$	$\mu^e = \frac{p}{\beta} - \Lambda + \alpha\Lambda\frac{\left(\mu^e - \frac{\Lambda}{2}\right)^2}{w}$	$S(\frac{\Lambda}{2}, \mu^e, p) \geq 0$
$\Lambda^{c1} < \Lambda < \Lambda^c$	$\mu^e = \frac{\Lambda}{2} + \frac{w}{v-p+\alpha\frac{\Lambda}{2}}$	$S(\frac{\Lambda}{2}, \mu^e, p) = 0$
$\Lambda \geq \Lambda^c$	$\mu^m = \lambda^m + \frac{w}{v-p+\alpha\lambda^m}$	$S(\frac{\Lambda}{2}, \mu^m, p) = 0$

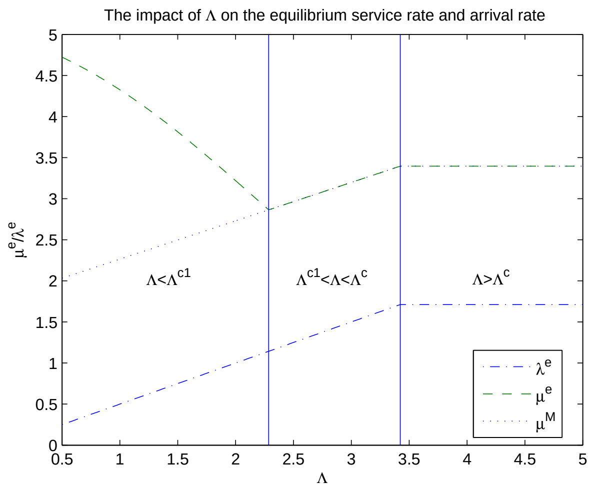

Fig. 2.6.3: The impact of the market size Λ on the service rate and the arrival rate in the symmetric equilibrium.

of the market, the service rate is already large even if consumer expected surplus is zero. Therefore, neither firm has motivation to increase the service rates. Thus, consumer expected surplus is zero in equilibrium.

Table 2.2 summarizes the impact of market size under social interactions on the equilibrium service rate and consumer expected surplus.

Fig.2.6.3 illustrates the impact of Λ on the equilibrium service rate. The parameters used in the numerical study are $v = 10$, $w = 9$, $\beta = 1$, $\alpha = 0.2$, $p = 5$.

From the figure, we can see when the total market size is small, each firm will determine a higher service rate, and consumers will get positive surplus in equilibrium. While when the total market size becomes large, consumer surplus will be reduced to their reservation level as in the monopoly case. Market size captures the competition intensity. A smaller market size leads to more intense competition among firms with substitutable services. Therefore, consumers can always get positive surplus in the market with more intense competition. The result may explain the situation in some service industries. For example, due to the small market, the competition in the telecommunication industry in Singapore is very intense, and internet service providers have to offer high speed services for their customers; while

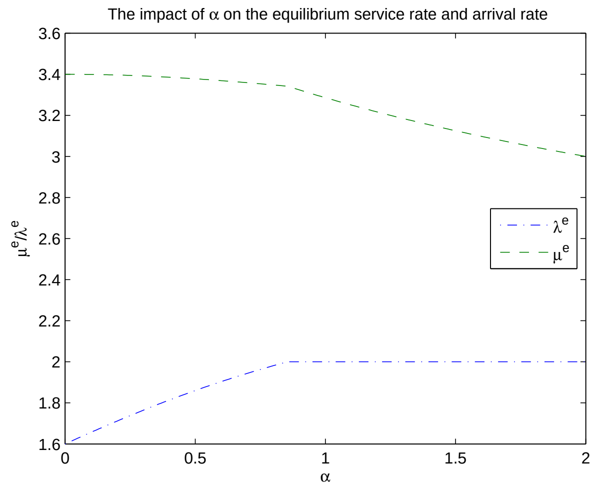

Fig. 2.6.4: The impact of the social interaction intensity α on the service rate and arrival rate in the symmetric equilibrium.

due to a huge market in China, the internet connection speed offered by all service providers is much lower than that in Singapore.

Since the term $\frac{3\beta\Lambda}{2} - p + \frac{\beta w(v-\alpha\frac{\Lambda}{2}-p)}{(v+\alpha\frac{\Lambda}{2}-p)^2}$ is decreasing in α, we conclude that given all the other parameters, for a fixed market size Λ, if α becomes large, consumers will benefit from social interactions, i.e., the expected surplus will be positive due to the increased service rate. Therefore, the influence of social interactions drives competing firms to decide a higher service rate, which reduces their expected profit while benefiting consumers. Fig.2.6.4 illustrates the impact of α on the equilibrium service rate. The following parameters are used $v = 10$, $w = 9$, $\beta = 1$, $\Lambda = 4$, $p = 5$.

2.7 Price competition under social interactions

In this section, we consider the two firms compete on the price decision while the service rate is assumed to be fixed. Since service rate can be considered as one aspect of service quality, the fixed service rate can be considered as the quality requirement regulated by authorities or industry standards, or the situation where the two firms are operated by one company with the same infrastructure.

2.7.1 Optimal Monopoly Price Under Social Interactions

In equilibrium, for a monopolist firm, consumer expected surplus must be zero under the optimal price decision. Therefore, the effective arrival rate satisfies

$$v + \alpha\lambda - CW(\lambda, \mu) - p = 0, \;\; \lambda \in [0, \Lambda]$$

where the optimal price will be $p = v + \alpha\lambda - \frac{w}{\mu-\lambda}$ if the effective arrival rate is λ.

In the following section, to reduce the complexity of the analysis, we assume $\mu > \Lambda$. The situation with $\mu \leq \Lambda$ can be analyzed similarly. Therefore, if the social interaction intensity is large enough, such that $\alpha \geq \frac{w}{(\mu-\Lambda)^2}$, p is always increasing in $\lambda \in [0, \Lambda]$; while if the social interaction intensity is small enough, such that$\alpha \leq \frac{w}{\mu^2}$, p is always decreasing in $\lambda \in [0, \Lambda]$; while if the social interaction intensity is intermediate, such that $\frac{w}{\mu^2} < \alpha < \frac{w}{(\mu-\Lambda)^2}$, p is first decreasing in $\lambda \in [0, \lambda^c]$, and then increasing in $(\lambda^c, \Lambda]$, where $\alpha = \frac{w}{(\mu-\lambda^c)^2}$. In the following section, we focus on the case where $\alpha \leq \frac{w}{\mu^2}$ is small, so that the effective arrival rate will decrease if the price increases.

The profit optimization problem of the monopolist is reformulated as

$$\max_{\lambda \in [0,\Lambda]} \pi(\lambda, \mu) = \max_{\lambda \in [0,\Lambda]} (v + \alpha\lambda - CW(\lambda, \mu) - \beta\mu)\lambda \tag{2.7.1}$$

and the optimal monopoly price decision is determined by the following result:

Lemma 2.5. *Given the service rate $\mu > \Lambda$, the optimal price will be $p^m = v + \alpha\lambda^m - \frac{w}{\mu-\lambda^m}$, where $\lambda^m = \min(\lambda^\star, \Lambda)$ is the optimal arrival rate, and $\lambda^\star$ satisfies the first order condition $\frac{\partial\pi(\lambda,\mu)}{\partial\lambda} = v + 2\alpha\lambda - \beta\mu - \frac{w\mu}{(\mu-\lambda)^2} = 0$.*

Proof. The first and second order derivatives of the profit function in terms of λ are given as

$$\frac{\partial\pi(\lambda,\mu)}{\partial\lambda} = v + 2\alpha\lambda - \beta\mu - \frac{w\mu}{(\mu-\lambda)^2}, \quad \frac{\partial^2\pi(\lambda,\mu)}{\partial\lambda^2} = 2\alpha - \frac{2w\mu}{(\mu-\lambda)^3} < 0$$

based on the assumption $\alpha \leq \frac{w}{\mu^2}$. Therefore, the optimal arrival rate will be $\lambda^m = \min(\lambda^\star, \Lambda)$, where $\lambda^\star$ satisfies the first order condition

$$\frac{\partial\pi(\lambda,\mu)}{\partial\lambda} = v + 2\alpha\lambda - \beta\mu - \frac{w\mu}{(\mu-\lambda)^2} = 0$$

□

Clearly, $\lambda^\star$ is increasing in α from the first order condition. Therefore, we can see, given the service rate μ, the optimal price and the optimal arrival rate depend on the social interaction intensity. If the social interaction intensity is large enough, the monopolist may cover the whole market by setting the corresponding price level. While if the social interaction intensity is small, such that $\lambda^\star < \Lambda$, the monopolist may only partially cover the market.

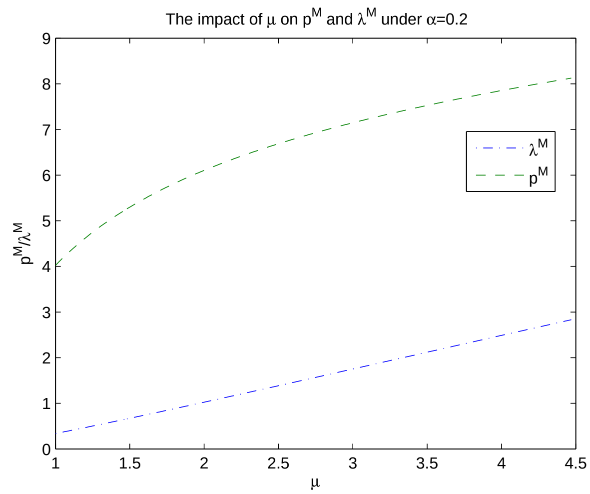

Fig. 2.7.1: The impact of the price p on the optimal service rate and arrival rate under the social interaction intensity $\alpha = 0.2$.

Given a fixed total market size Λ, Fig.2.7.1-2.7.2 illustrate the impact of service rate given fixed social interaction intensity, and the impact of social interaction intensity given fixed service rate on the optimal price decision in the monopoly setting. The following parameters are used in the numerical example, $v = 10$, $w = 4$, $\beta = 1$.

Therefore, if the service rate or social interaction intensity increases, more market will be covered, and the price will also increase in the monopolist setting. In other words, given the fixed service rate, social interactions can help firms to charge a higher price and cover a large market, thus lead to more profit.

2.7.2 Price Competition Under Social Interactions

From the optimal price decision for the monopolist, we can see if

$$v + \alpha\Lambda - \beta\mu - \frac{w\mu}{(\mu - \frac{\Lambda}{2})^2} \leq 0$$

the optimal monopoly arrival rate is $\lambda^m \leq \frac{\Lambda}{2}$. Therefore, service competition does not impact the price decision and each service provider will be operated as a monopolist with the optimal price charged at $p^m = v + \alpha\lambda^m - \frac{w}{\mu - \lambda^m}$. The LHS of the above condition is decreasing in Λ, since $\alpha - \frac{w\mu}{(\mu-\frac{\Lambda}{2})^3} < \alpha - \frac{w}{(\mu-\frac{\Lambda}{2})^2} < \alpha - \frac{w}{\mu^2} \leq 0$ based on the assumption. Therefore, if Λ is large enough, such that $\Lambda \geq \Lambda^c$, where Λ^c satisfies the equation $v + \alpha\Lambda^c - \beta\mu - \frac{w\mu}{(\mu-\frac{\Lambda^c}{2})^2} = 0$, each firm will be operated as a monopolist.

In the following section, we focus on the case $\Lambda < \Lambda^c$, where the monopoly

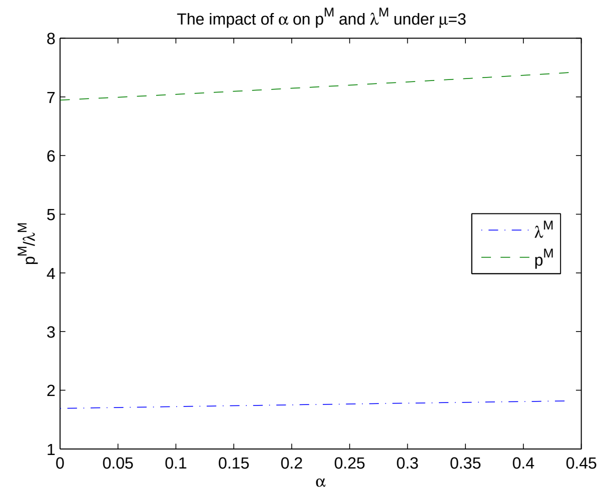

Fig. 2.7.2: The impact of the social interaction intensity α on the optimal service rate and arrival rate under the service rate $\mu = 3$.

optimal arrival rate is $\lambda^m > \frac{\Lambda}{2}$. Therefore, the competition impacts firm pricing decisions. Similar to the previous section, the market will be fully covered, and the firm with a larger price will cover a smaller market, as shown in the following result:

Lemma 2.6. *If $\Lambda < \Lambda^c$, the market will be fully covered. If $p_1 \geq p_2$, the effective arrival rate satisfies $\lambda_1 \leq \frac{\Lambda}{2} \leq \lambda_2$ and $\lambda_1 + \lambda_2 = \Lambda$.*

Proof. It is straightforward that the market will be fully covered. Since $\alpha \leq \frac{w}{\mu^2}$, given any price p, the surplus $S(\lambda, p) = v + \alpha\lambda - CW(\lambda, \mu) - p$ is decreasing in $\lambda \in [0, \Lambda]$. Suppose $\lambda_1 < \lambda_2$ if $p_1 < p_2$. We have $S(\lambda_2, p_2) < S(\lambda_1, p_2) \leq S(\lambda_1, p_1)$ where the first inequality follows from the decreasing property of $S(\lambda, p)$ with respect to λ, and the second inequality follows since $S(\lambda, p)$ is increasing in p for fixed λ. Therefore, $\lambda_1 < \lambda_2$ can not be equilibrium. □

Therefore, under the fixed service rate constraint, a smaller price will always attract a larger effective arrival rate if the social interaction intensity is small. We consider the symmetric equilibrium, where each firm sets the price at $p_i = p^e$ and covers $\lambda_i = \frac{\Lambda}{2}$. Clearly, in the symmetric equilibrium, $p^e \leq v + \alpha\frac{\Lambda}{2} - \frac{w}{\mu - \frac{\Lambda}{2}}$ since consumer expected surplus can not be negative. In the following section, we investigate whether $p^e = v + \alpha\frac{\Lambda}{2} - \frac{w}{\mu - \frac{\Lambda}{2}}$ can be an equilibrium under service competition. Similar to the result in service rate competition, market size plays an important role in equilibrium price decisions. If the total potential market size is intermediate, we have the following result:

Proposition 2.4. *Under the assumption $\alpha \leq \frac{w}{\mu^2}$, if the market size is in the range*

$(\Lambda^{c1}, \Lambda^c)$, where Λ^{c1} satisfies the following condition

$$v + \frac{3}{2}\alpha\Lambda^{c1} - \beta\mu - \frac{w(\mu + \frac{\Lambda^{c1}}{2})}{(\mu - \frac{\Lambda^{c1}}{2})^2} = 0$$

$p^e = v + \alpha\frac{\Lambda}{2} - \frac{w}{\mu - \frac{\Lambda}{2}}$ is a symmetric equilibrium. If the following condition holds,

$$\left(v + \alpha\frac{\Lambda}{2} - \frac{w}{\mu - \frac{\Lambda}{2}} - \beta\mu\right)\frac{(\mu - \frac{\Lambda}{2})^2}{2w - 2\alpha(\mu - \frac{\Lambda}{2})^2} - \frac{\Lambda}{2} \leq 0$$

and

$$\forall\delta \in [0, p^e - \beta\mu],\ (p^e - \beta\mu - 2\delta)\epsilon''(\delta) - 4\epsilon'(\delta) \leq 0$$

where ϵ satisfies $F(\delta, \epsilon) = (2\alpha\epsilon + \delta)\left((\mu - \frac{\Lambda}{2})^2 - \epsilon^2\right) - 2w\epsilon = 0$, $\epsilon'(\delta)$ and $\epsilon''(\delta)$ are the first and second order conditions for ϵ in terms of δ, then the symmetric equilibrium price p^e is the unique Nash equilibrium.

If the total market size is small enough, we have the following result:

Proposition 2.5. *Under the assumption $\alpha \leq \frac{w}{\mu^2}$, if the market size satisfies $\Lambda \leq \Lambda^{c1}$, there exists a symmetric equilibrium in terms of the price competition, where each firm covers one half of the market by setting the price at*

$$p^e = \beta\mu + \frac{4w\Lambda}{(2\mu - \Lambda)^2} - \alpha\Lambda$$

and consumer expected surplus is strictly non-negative. If the following condition holds,

$$\forall\delta \in [0, p^e - \beta\mu],\ (p^e - \beta\mu - 2\delta)\epsilon''(\delta) - 4\epsilon'(\delta) \leq 0$$

where ϵ satisfies $F(\delta, \epsilon) = (2\alpha\epsilon + \delta)\left((\mu - \frac{\Lambda}{2})^2 - \epsilon^2\right) - 2w\epsilon = 0$, $\epsilon'(\delta)$ and $\epsilon''(\delta)$ are the first and second order conditions for ϵ in terms of δ, then the equilibrium price p^e is the unique Nash equilibrium.

From $\beta\mu + \frac{4w\Lambda}{(2\mu-\Lambda)^2} - \alpha\Lambda = v + \alpha\frac{\Lambda}{2} - \frac{w}{\mu - \frac{\Lambda}{2}}$, we can get the condition $v + \frac{3}{2}\alpha\Lambda - \beta\mu - \frac{w(\mu + \frac{\Lambda}{2})}{(\mu - \frac{\Lambda}{2})^2} = 0$. Therefore, if $\Lambda \leq \Lambda^{c1}$, $\beta\mu + \frac{4w\Lambda}{(2\mu-\Lambda)^2} - \alpha\Lambda \leq v + \alpha\frac{\Lambda}{2} - \frac{w}{\mu - \frac{\Lambda}{2}}$. The reason is as follows. If the total potential market size is small enough, the competition is more intense. Each firm wants to cover as much of the market as possible. Therefore, both firms will charge a lower price which leads to a strictly

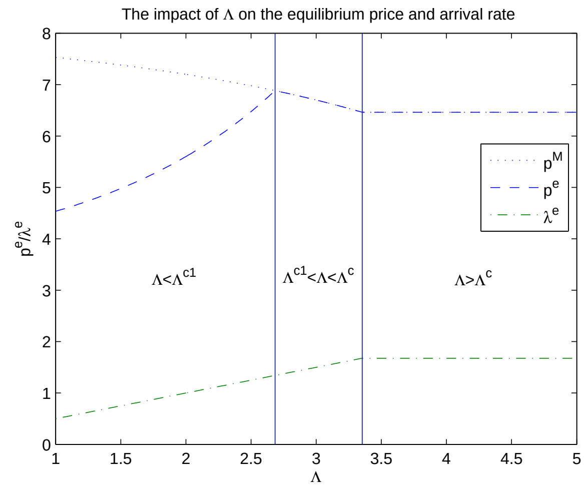

Fig. 2.7.3: The impact of the market size Λ on the service rate and arrival rate in the symmetric equilibrium.

positive expected surplus for consumers. If the total potential market becomes large, competition becomes less intense. Each firm can charge a higher price to reduce consumer expected surplus to zero and covers one half of the market. Neither firm has motivation to lower their prices. Thus, consumer expected surplus is zero in equilibrium. The result may also explain the situation in some service industries. Taking the telecommunication industry as an example, the market size is so small such that the competition intensity becomes extremely strong, and the service providers have to charge a lower price to their customers, which is a sharp contrast to the price charged by China telecommunication firms, since there is a huge market in China.

Fig.2.7.3 illustrates the impact of Λ on the equilibrium service rate. The following parameters are used in the numerical study, $v = 10$, $w = 9$, $\beta = 1$, $\alpha = 0.2$, $\mu = 4$.

Since the term $v + \frac{3}{2}\alpha\Lambda - \beta\mu - \frac{w(\mu+\frac{\Lambda}{2})}{(\mu-\frac{\Lambda}{2})^2}$ is increasing in α, we conclude that given all the other parameters are fixed, if α becomes large, consumers will benefit from social interactions, i.e., the expected surplus will be positive, since the equilibrium price will become smaller. The larger the social interaction intensity is, the smaller the price will be. Therefore, the influence of social interaction drives competing firms to charge a lower price, which reduces their expected profit while benefiting consumers. The equilibrium price under a small market also indicates, each firm will not charge the minimum price while still getting strictly a positive profit in equilibrium, since $\frac{4w\Lambda}{(2\mu-\Lambda)^2} > \alpha\Lambda$. However, compared with the case without social interactions ($\alpha = 0$), social interactions always lead to a smaller price which reduces firm profit under a small market $\Lambda \leq \Lambda^{c1}$. Therefore, under a small market with more intense competition, social interactions may harm firms in terms of profit, since

Tab. 2.3: The impact of market size on the equilibrium price and consumer expected surplus under social interactions.

Market size	Equilibrium price	Consumer expected surplus
$\Lambda \leq \Lambda^{c1}$	$p^e = \beta\mu + \frac{4w\Lambda}{(2\mu-\Lambda)^2} - \alpha\Lambda$	$S(\frac{\Lambda}{2}, \mu, p^e) \geq 0$
$\Lambda^{c1} < \Lambda < \Lambda^c$	$p^e = v + \alpha\frac{\Lambda}{2} - \frac{w}{\mu-\frac{\Lambda}{2}}$	$S(\frac{\Lambda}{2}, \mu, p^e) = 0$
$\Lambda \geq \Lambda^c$	$p^m = v + \alpha\lambda^m - \frac{w}{\mu-\lambda^m}$	$S(\frac{\Lambda}{2}, \mu, p^m) = 0$

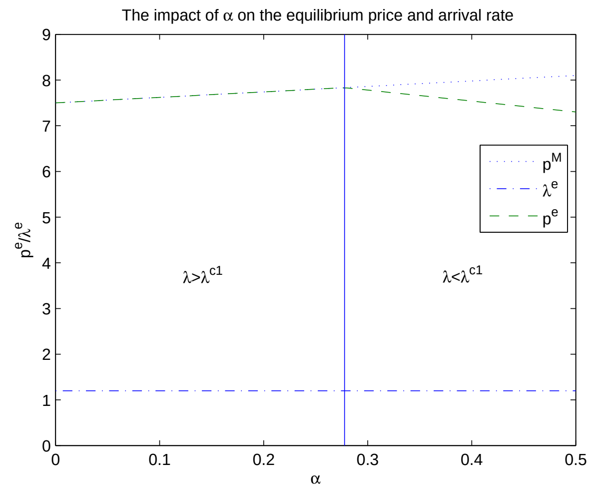

Fig. 2.7.4: The impact of the social interaction intensity α on the service rate and arrival rate in the symmetric equilibrium.

a lower price has to be charged.

Table 2.3 summarizes the impact of market size under social interactions on the equilibrium price and consumer expected surplus.

Fig.2.7.4 illustrates the impact of α on the equilibrium price decision. The following parameters are used in the numerical study $v = 10$, $w = 9$, $\beta = 1$, $\Lambda = 4$, $p = 5$.

2.8 Service Rate and Price Competition

Since there are two operational decisions, to fully characterize the equilibrium decisions is not easy. In this section, we consider the service rate and price competition simultaneously under the condition that each firm adopts the *fixed profit margin* policy. The profit margin is defined as $\Delta = p - \beta\mu$, i.e., the net profit from each served consumer. In some service industries, the marginal profit may be strictly regulated by public policies or industry standards. Through the fixed profit margin constraint, the two-dimensional decision problem is reduced as a one-dimensional decision problem, either the service rate or the price. In the following section, we focus on the service rate decision in the competition. Since the profit margin is fixed, we assume given the effective arrival rate, each service provider will set the service rate and the corresponding price to maximize consumer expected surplus.

2.8.1 Monopoly Service Rate and Price with Fixed Profit Margin

For a monopolist with the profit margin Δ, given the service rate $\mu > 0$, the price will be $p = \beta\mu + \Delta$. In equilibrium, consumer expected surplus denoted as $S(\lambda, \mu)$ must be non-negative as

$$S(\lambda, \mu) = v + \alpha\lambda - CW(\lambda, \mu) - \beta\mu - \Delta \geq 0,\ \ \lambda \in [0, \Lambda]$$

The monopolist maximizes the profit $\pi(\mu) = \Delta\lambda$ on $\lambda \in [0, \Lambda]$ by choosing the corresponding service rate μ. We assume $v - \Delta \geq 0$ and $(v - \Delta)^2 - 4\beta w \geq 0$, so that the available range of the service rate is non-empty.

Since the profit margin is fixed, the monopolist wants to cover the market as much as possible. However, it may not be possible for the monopolist to cover the whole market. If the firm covers the whole market, the service rate would be larger, and the price will be larger, which reduces consumer expected surplus. Therefore, given the arrival rate $\lambda \in [0, \Lambda]$, we first calculate the maximum surplus that the monopolist can provide to the consumers as

$$S^M(\lambda) = \max_{\mu} S(\lambda, \mu) = v + (\alpha - \beta)\lambda - 2\sqrt{\beta w} - \Delta$$

where $\mu = \lambda + \sqrt{\frac{w}{\beta}}$. The curve $S^M(\lambda) = S(\lambda, \lambda + \sqrt{\frac{w}{\beta}}),\ \lambda \in [0, \Lambda]$ is the maximum expected surplus if the effective arrival rate is λ. We have the following result in terms of the monopoly optimal service rate under the fixed profit margin policy:

Lemma 2.7. *Given the fixed profit margin Δ, the monopoly optimal service rate is given in the following cases:*

- *If $\alpha \geq \beta$, the optimal service rate will be in the range*

$$\begin{aligned} \mu^m \ \in \ & [\frac{v + (\alpha + \beta)\Lambda - \Delta - \sqrt{(v + (\alpha - \beta)\Lambda - \Delta)^2 - 4\beta w}}{2\beta}, \\ & \frac{v + (\alpha + \beta)\Lambda - \Delta + \sqrt{(v + (\alpha - \beta)\Lambda - \Delta)^2 - 4\beta w}}{2\beta}] \end{aligned}$$

  and the monopolist covers the whole market, i.e., $\lambda^m = \Lambda$;

- *If $\alpha < \beta$, we have the following cases:*

– *If* $v + (\alpha - \beta)\Lambda - 2\sqrt{\beta w} - \Delta \geq 0$*, the optimal service rate will be the same as in the above range and the monopolist covers the whole market* $\lambda^m = \Lambda$*;*

– *If* $v+(\alpha-\beta)\Lambda-2\sqrt{\beta w}-\Delta < 0$*, the optimal service rate will be* $\mu^m = \lambda^m + \sqrt{\frac{w}{\beta}}$*, and the monopolist partially covers the market as* $\lambda^m = \frac{v-2\sqrt{\beta w}-\Delta}{\beta-\alpha}$*.*

The above result indicates, if the social interaction intensity is large, such that $\alpha \geq \beta$, the monopolist can always cover the whole market by setting appropriate service rate and price for any fixed profit margin level. Since $\alpha \geq \beta$ and the profit margin is fixed, suppose the arrival rate increases from λ to $\lambda + \delta$. The service rate can be increased from $\mu + \delta$, meanwhile consumer expected surplus increases, since the congestion cost keeps the same. Therefore, the market will always be fully covered for any fixed profit margin. However, if the social interaction intensity is small, the firm may not cover the whole market. The above result also confirms the optimal price and capacity decisions for the monopolist firm in the previous section. The optimal profit margin Δ can be derived which is omitted here.

2.8.2 Service Rate and Price Competition Under Social Interactions

From the above result, we can see if $\alpha < \beta$ and the total potential market size Λ is large enough, such that

$$\frac{v - 2\sqrt{\beta w} - \Delta}{\beta - \alpha} \leq \frac{\Lambda}{2},$$

each firm's optimal decision is to partially cover the market, which is smaller than one half of the market, under the fixed profit margin $\Delta \geq 0$. Therefore, the competition does not impact the service rate and price decisions.

While if $\alpha \geq \beta$, the service rate decision will be impacted by the competition between the two firms, since each firm's optimal decision is to cover the whole market under the fixed premium $\Delta \geq 0$. If $\alpha < \beta$ and the total potential market Λ is small, such that

$$\Lambda \leq \frac{v - 2\sqrt{\beta w} - \Delta}{\beta - \alpha},$$

each firm's optimal decision is still to cover the whole market under the fixed profit margin $\Delta \geq 0$. The competition between the two firms will impact the service rate decision.

We first consider the case $\alpha \geq \beta$ in the following section. Similar to previous sections, the market will be fully covered in equilibrium. Since the premium is fixed, each firm will try to cover the market as much as possible. We have the following result:

Lemma 2.8. *If $\alpha \geq \beta$, there exists a Nash equilibrium in terms of service rate decision, where each firm sets $\mu^e = \Lambda + \sqrt{\frac{w}{\beta}}$. In the Nash equilibrium, the effective arrival rate will be $(\Lambda, 0)$ or $(0, \Lambda)$, i.e., only one firm covers the whole market.*

The above result indicates, if the social interaction intensity is strong, such that $\alpha \geq \beta$, each firm will set the service rate to maximize consumer expected surplus at the level such that all consumers will join the same queue. However, all the consumers will only join one queue in equilibrium. Therefore, a large social interaction intensity will always benefit consumers. From a long-run perspective, if each period starts anew, each firm will cover $\frac{\Lambda}{2}$ on average, i.e., consumers may select the service provider alternatively.

We consider the case where $\alpha < \beta$ and $\Lambda \leq 2\frac{v-2\sqrt{\beta w}-\Delta}{\beta-\alpha}$, where each firm can cover more than one half of the market as a monopolist, i.e., $S(\frac{\Lambda}{2}) > 0$. Since $\alpha < \beta$, if all consumers join the same queue, the maximum surplus $S(\Lambda, \Lambda + \sqrt{\frac{w}{\beta}})$ is the smallest. We first have the following result:

Lemma 2.9. $S(\lambda, \mu) \leq S(\mu - \sqrt{\frac{w}{\alpha}}, \mu) \leq S(\mu - \sqrt{\frac{w}{\alpha}}, \mu - \sqrt{\frac{w}{\alpha}} + \sqrt{\frac{w}{\beta}})$.

Proof. The first inequality follows from the fact that given μ, consumer expected surplus is maximized at $\lambda = \mu - \sqrt{\frac{w}{\alpha}}$ due to the concavity of $S(\lambda, \mu)$ in terms of λ. The second inequality follows from the fact that given λ, consumer surplus is maximized at $\mu = \lambda + \sqrt{\frac{w}{\beta}}$ due to the concavity of $S(\lambda, \mu)$ in terms of μ. □

The above result indicates, given λ and μ, the maximum consumer expected surplus is $S(\lambda, \lambda + \sqrt{\frac{w}{\beta}})$, i.e., $\forall \lambda \in [0, \Lambda]$, $S^M(\lambda) = S(\lambda, \lambda + \sqrt{\frac{w}{\beta}})$, in the sense that $S(\lambda)$ crosses the maximum point of $S(\lambda, \mu)$ for fixed μ. For example, we can compare the following surplus $S(\frac{\Lambda}{2}, \frac{\Lambda}{2} + \sqrt{\frac{w}{\beta}})$ and $S(\frac{\Lambda}{2}, \frac{\Lambda}{2} + \sqrt{\frac{w}{\alpha}})$ as

$$S(\frac{\Lambda}{2}, \frac{\Lambda}{2} + \sqrt{\frac{w}{\beta}}) - S(\frac{\Lambda}{2}, \frac{\Lambda}{2} + \sqrt{\frac{w}{\alpha}}) = \sqrt{\alpha w} + \beta\sqrt{\frac{w}{\alpha}} - 2\sqrt{\beta w} \geq 0$$

We focus on the symmetric equilibrium, where each firm sets the service rate at $\mu = \mu^e$ and covers one half of the market, i.e., $\lambda^e = \frac{\Lambda}{2}$. In the following section, we investigate whether the service rate $\mu^e = \frac{\Lambda}{2} + \sqrt{\frac{w}{\beta}}$ can be the equilibrium. It is straightforward that, consumer expected surplus will be maximized at μ^e given one half of the consumers join the same queue.

We assume the following condition holds:

$$S(0, \frac{\Lambda}{2} + \sqrt{\frac{w}{\beta}}) > v + (\alpha - \beta)\Lambda - 2\sqrt{\beta w} - \Delta$$

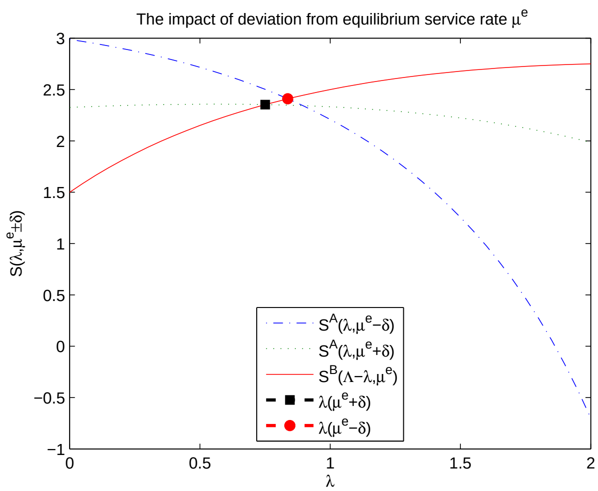

Fig. 2.8.1: The impact of the deviation from the equilibrium service rate.

which indicates if no consumers join the queue with the service rate $\frac{\Lambda}{2}+\sqrt{\frac{w}{\beta}}$, consumer expected surplus is larger than the maximum surplus from the other queue if all the consumers join there. Graphically, in the range $\lambda \in (\frac{\Lambda}{2}, \Lambda]$, the consumer expected surplus curve $S(\Lambda-\lambda, \frac{\Lambda}{2}+\sqrt{\frac{w}{\beta}})$ is always above the maximum surplus curve $S(\lambda)$ for $\lambda \in [0, \Lambda]$. Based on the above assumption, we have the following result:

Proposition 2.6. *If $\alpha < \beta$ and $\Lambda \leq 2\frac{v-2\sqrt{\beta w}-\Delta}{\beta-\alpha}$, there exists a unique equilibrium in terms of service rate decision, where each firm sets $\mu^e = \frac{\Lambda}{2}+\sqrt{\frac{w}{\beta}}$ and covers $\lambda^e = \frac{\Lambda}{2}$, providing the following condition is satisfied: $S(0, \frac{\Lambda}{2}+\sqrt{\frac{w}{\beta}}) > v+(\alpha-\beta)\Lambda - 2\sqrt{\beta w}-\Delta$.*

Therefore, we can see if the surplus curve $S(\Lambda-\lambda, \frac{\Lambda}{2}+\sqrt{\frac{w}{\beta}})$ is above the maximum surplus curve $S^M(\lambda)$ in the range $\lambda \in (\frac{\Lambda}{2}, \Lambda]$, the service rate $\mu^e = \frac{\Lambda}{2}+\sqrt{\frac{w}{\beta}}$ is the unique symmetric Nash equilibrium, where consumer surplus is maximized and exactly half of the market will be covered by each firm. If the above condition is not satisfied, we may have multiple equilibria as discussed in Brock and Durlauf (2001).

Fig.2.8.1 illustrates the impact of the deviation in service rate decision from the equilibrium level on the effective arrival rate, given the other firm chooses the equilibrium service rate. The parameters used in the example are $v = 10$, $w = 9$, $\alpha = 0.5$, $\beta = 1$, $\Lambda = 2$, $\Delta = 1$. Therefore, from the figure, we can see if the other firm chooses the equilibrium service rate, a larger or smaller service rate will result in a lower arrival rate, which indicates deviation from the equilibrium service rate is always suboptimal.

Fig.2.8.2 illustrates the equilibrium service rate and consumer expected surplus

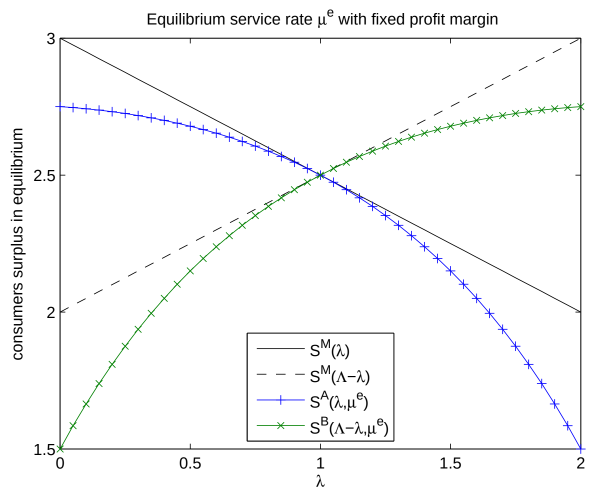

Fig. 2.8.2: The equilibrium service rate μ^e and consumer expected surplus under the fixed profit margin policy.

under the fixed profit margin policy with the parameters $v = 10, w = 9, \alpha = 0.5, \beta = 1, \Lambda = 2, \Delta = 1$.

2.9 Conclusion

The influence of social interactions is significant in consumer purchase decision-making, which, as an additional driving force, leads to more intense competition in the consumer market and severe challenges in operations management. How operational decisions should be adapted and implemented under social interactions in the competitive market becomes critical for firms to survive and succeed.

The influence of social interactions, which can be a double-edged sword, brings both opportunities and challenges in operations. Social interactions may help firms attract more demand to achieve market expansion and revenue growth, since consumers become more inclined to purchase products and services which are also preferred by others. However, increased demand through social interactions may lead to a more congested service system due to capacity constraint which is costly to expand. Congestion increases consumer waiting cost, which may cause potential revenue loss. Thus, social interactions create both positive and negative externalities in consumer purchase decisions. Since consumers are more likely to make the same purchase decision as others do, competition among firms for market share may become even more intense under social interactions. Thus, competition and social interactions as well as their interplay demand for appropriate operational strategies to improve firm performance.

In this chapter, we consider service competition in terms of price and capacity decisions between two homogeneous firms in a fixed size consumer market under social interactions. By focusing on the symmetric equilibrium, we investigate the impact of social interactions on the equilibrium capacity and price decisions. The result suggests that in the monopoly market, social interactions can always benefit the firm, where a large market can be covered while a smaller capacity is adopted and a higher price is charged. More influential social interactions may help the monopolist cover the whole market. If managers ignore social interactions in decision-making, a substantial potential market and revenue will be lost due to inefficient price and capacity decisions.

However, in competition, social interactions may not always benefit firms, especially when the social interaction intensity is large and the market size is small. Under a small market, due to competition, firms have to invest in large capacities or charge low prices, while social interactions compel firms to build even more capacities or charge even lower prices. Therefore, social interactions under a small market may always drive firms to sacrifice more profit in competition. If the social interaction intensity becomes larger, firms need to build more capacities and charge lower prices which will lead to less profit in competition. Consumers may always benefit from more intense social interactions with positive surplus from consumption; therefore, the presence of social interactions may harm firms while benefiting consumers in a small competitive market. However, under a large market, if the social interaction intensity is small, each firm can still benefit from social interactions, where the price and the service rate can be determined at the level where consumer expected surplus is reduced as low as their reservations.

Therefore, if managers ignore the influence of social interactions in consumer purchase decisions in the competitive market, they may over-price the products or services, build insufficient capacities when the market size is small; they may under-price their products or services, over-invest in capacities when the market size is large. Therefore, ignoring social interactions, potential profit will be lost in the consumer market with fierce competition. As discussed, market size captures the competition intensity. Our result indicates that social interactions may drive firms to charge a lower price, build a larger capacity to provide a more responsive service, since competition intensity may be aggravated due to changes in consumer preferences and purchase behaviors. Be aware of social interactions and their interplay with competition will help managers better improve their operational decisions and firm performance.

The model in this chapter suffers from several limitations. We use M/M/1 queue to model the operations of the service provider for analytical tractability. As discussed in this chapter, although an M/M/1 queue captures the congestion phenomenon for many service systems, the Poisson arrival rate assumption can not capture the impact of variability of the arrival rate to the system performance. Therefore, one potential extension of the current model is to study the impact of social interactions based on the G/M/1 queueing system. In this chapter, we mainly focus on the impact of positive social interaction effect on the operational decisions. The model can be generalized to the case with negative social interactions, and some key results can be derived. For example, in service competition, compared with the case where social interactions are absent, under a small market, the negative social interaction effect may offset the impact of competition, where firms can charge a higher price or build a lower capacity; while under a large market, each firm may suffer from the negative social interaction effect, where a lower price or a higher capacity would be optimal. In this chapter, we consider a linear form of social value for analytical tractability. Some non-linear features of social interactions may be lost due to a linear form. A more realistic functional form may be a convex-concave function, where the influence of social interactions depends on the number of consumers. However, the close form solution in terms of equilibrium price and capacity decisions may not be easy to derive. We also assume the social interaction intensity is a constant, which is exogenously given. In reality, social interactions may also depend on operational decisions, such as price. For example, services with a lower price may help attract more potential consumers. The optimal price and service rate decisions under price-dependent social interaction intensity is listed in the Appendix 6.1.

3. DYNAMIC CAPACITY AND PRICE DECISIONS UNDER SOCIAL INTERACTIONS

3.1 Abstract

Under social interactions, consumers perceive a higher value of a product or service with more existing adopters. Social interactions help firms attract more potential demand, and also lead to more congestion, which bring challenges in operations. To investigate how social interactions impact operations in profit optimization and market expansion, we focus on price and capacity decisions of a monopolist firm in a market with repeated interactions. Due to capacity constraint, managers need to balance the social interaction effect and congestion effect through price and capacity decisions. The firm is operated as an M/M/1 queueing system, and the capacity of the firm is determined by the service rate. Consumers are rational in queue-joining decisions. Both strategic and myopic policies in terms of price and capacity are investigated and compared under several dynamic settings.

Under several dynamic settings, the results indicate that price and service rate decisions lead to monotonic arrival rate paths, which converge to a unique steady state. Strategic policies always achieve a larger market. Under strategic policies, a lower price or a higher capacity should be adopted to build up a larger customer base which helps attract more potential consumers through social interactions. If price and capacity are simultaneously determined, a lower profit margin may be desired to increase the customer base in the strategic policy although both higher prices and capacities are adopted than those in the myopic policy. Profit margins in strategic policies may always be smaller than those in myopic policies, especially when social interactions are less intense. However, strategic policies can always achieve a larger profit after initial periods when a larger market base has been established. Under social interactions, managers should tradeoff short term profit loss for long term benefit in operational decision-making. Ignoring such long term implications of their operational decisions, managers may charge inappropriately high prices, build insufficient capacities, or grab too high profit margins. Be aware of social interactions; operational decisions such as price and capacity can be better determined to improve firm performance.

3.2 Introduction

In order to enter into the bestseller list, two authors secretly purchased 50,000 copies of their book *The Discipline of Market Leaders* from various stores whose sales were monitored to select books for the New York Times bestseller list[1]. Despite mediocre reviews, that book entered into the bestseller list and subsequently sold well enough to continue as a bestseller without further *demand intervention* (Bikhchandani et al., 1998). Recently, in order to boost sales performance of their flagship-brand cars in the US, BMW AG adopted *sales inflation* by counting many demo models in a sales announcement[2]. Many similar cases have been reported that demand intervention and sales inflation are often intentionally or secretly adopted by companies in order to promote their products into bestseller lists or top in sales rankings. The fact that companies take such tactics, even *unethically*, highlights the reality of the influence of *social interactions*.

Due to social interactions, consumers are inclined to purchase those products or services with large existing sales, or followed by long queues. Herding in long queues is a typical phenomenon under the influence of social interactions, which drives consumers to join a long queue for a popular restaurant (Becker, 1991), wait for many hours in order to see a general practitioner[3], etc. The significant influence of social interactions brings opportunities for companies to increase profitability and expand market share. However, social interactions also raise challenges in operations, such as price and capacity decisions. On the one hand, social interactions help attract more potential demand which helps firms increase sales and achieve fast growth. On the other hand, increased demand may create congestion leading to an intolerable waiting experience for consumers due to insufficient capacities which are usually costly for firms to expand. Capacity constraint often reduces flexibility in operations and restricts firm capabilities in product and service delivery, often leading to congestion, which may be aggravated due to excess demand induced by social interactions. Congestion due to insufficient capacities leads to potential revenue loss, since consumers may never come back again for dining[4], healthcare services[5], or even grocery shopping[6]. Therefore, better understanding of the impact of social interactions on operational decisions is required for managers to improve operational decision-making to achieve profitability and maintain stable growth.

Under social interactions, poorly managed operations, such as an inappropriate price or capacity decision, may lead to demand fluctuations, potential customer and

[1] http://www.businessweek.com/stories/1995-08-06/did-dirty-tricks-create-a-best-seller
[2] http://online.wsj.com/article/SB10000872396390444042704577589511359646688.html
[3] http://www.asianewsnet.net/news-34738.html
[4] http://www.nytimes.com/2010/06/09/dining/09reservations.html?pagewanted=all
[5] http://online.wsj.com/article/SB10001424052702304410504575560081847852618.html
[6] http://www.nytimes.com/2007/06/23/business/23checkout.html

revenue loss. It has been reported that due to huge demand and insufficient supply due to poor capacity planning and supply chain management, consumers have had to wait several weeks for the new smart phone Nexus 4[7], and substantial potential revenue may have been lost. Therefore, under social interactions, how management practices and operational decisions should be adapted and implemented becomes critical for firms to achieve sustainable growth and profitability.

To investigate the impact of social interactions on operational decisions, we consider the long-term capacity and price decisions for a profit maximizing firm providing frequently purchased product or service in the consumer market. For exposition simplicity, we assume a service is offered. Service is a broad term in this chapter, which may refer to specific services offered by service providers, or the products supplied by make-to-order firms, etc. We adopt *social interaction effect* to characterize the phenomenon that *potential consumers* are more inclined to purchase the service when more *existing consumers* are present. The firm is operated as a queueing system to capture the congestion effect due to capacity constraint. Under several dynamic settings, both strategic and myopic policies in terms of price, capacity, and joint price and capacity are investigated and compared. Based on the optimal pricing and capacity strategies derived, this study aims to offer some possible explanations or reasons for several interesting phenomena observed in practice, such as (1) why firms with popular products or services do not leverage on the popularity to raise prices; (2) why some firms can charge higher prices or profit margins than their competitors even if similar or the same service is offered; (3) why some firms suffer from significant loss of market share after changing their prices; (4) why some firms charge seemingly unreasonably low prices or build seemingly excessive capacities.

The chapter is organized as follows. In Section 3.3, we briefly review several studies which focus on the influence of social interactions in consumer purchase decisions. We discuss the settings of the model in Section 3.4. Section 3.5 discusses the dynamic model with constant service rate and price. Section 3.6 investigates the capacity/service rate decisions, where the price is fixed. Section 3.7 focuses on the dynamic pricing model with fixed service rate. In Section 3.8, we investigate the joint capacity and price decisions under social interactions. Section 3.9 concludes this chapter with summary and managerial insights.

3.3 Related Studies

The influence of social interactions in individual decision-making has been investigated extensively, see the brief review in Chapter 1. Similar to Chapter 2, in this chapter, the influence of social interactions studied can be due to the reference group

[7] http://www.ibtimes.com/nexus-4-sold-out-again-us-google-ceo-says-supply-shortages-are-priority-nexus-team-1042598#

effect, OL, WOM communication or network externality. There are extensive studies on the influence of social interactions in consumer purchase decisions including both empirical investigations and theoretical models.

Empirical investigations focus on identifying and measuring the influence of social interactions. Using a natural experiment, Kraut et al. (1998) find that due to social interactions, employees will adopt a particular video system with more existing users for both utility and normative reasons. Similarly, Manchanda et al. (2008) and Angst et al. (2010) find significant influence of social interactions on adoption of new products in the pharmaceutical industry and the diffusion of electronic medical records in U.S. hospitals. The influence of social interactions not only increases household participation in the stock-market (Hong et al., 2004), but also drives employees to choose the same healthcare plans (Sorensen, 2006). People's satisfaction judgments can also be modified due to social interactions (Bohlmann et al., 2006). Hartmann (2010) estimates the influence of social interactions on an individual's utility based on the purchase decisions within several golf players. The findings indicate around 35% of the median consumer value is attributable to the influence of social interactions with their peers. In the study of Moretti (2011), 32% of sales for a typical movie with positive surprise is attributed to social interactions, based on the box-office data for all movies released between 1982 and 2000, where consumers consider sales data as an indicator of quality.

Consumer purchase decision has been influenced substantially through various online social interactions in recent years. As consumers become overloaded with information, they become increasingly skeptical about traditional company-driven advertising and marketing while increasingly prefer information from social interactions, such as product reviews (Chevalier and Mayzlin, 2006; Chen et al., 2011), recommendations through WOM, and OL through previous sales information. Surveyed by Mckinsey Quarterly, 64% of respondents said that WOM influenced their purchase decisions (Atsmon et al., 2011). WOM has become one critical factor that influences a firm's market share, which can be increased by 10% or reduced by 20% due to the pass-on rates for key positive and negative messages over a two-year period (Bughin et al., 2010). WOM can even help newly established firms to survive and succeed in the competitive market. The exponential acceleration in market share of Google was mostly due to WOM for the first few years of its existence (Moretti, 2011). Through OL, previous sales performance, such as the number of downloads of a particular software, the number of bids of an auction, can significantly influence potential consumer choices (Hanson and Putler, 1996; Onnela and Reed-Tsochas, 2009; Simonsohn and Ariely, 2008). Through a natural experiment, Cai et al. (2009) find that when customers are given ranking information of the five most popular dishes, the demand for those dishes increases by 13 to 20 percent due to OL. In online transactions, the popularity information can shape consumer choices substantially, such as the wedding service vendor selection (Tucker and Zhang, 2011), and the lending decisions to finance potential borrowers (Zhang and Liu, 2012), where lenders always draw quality information from observing others' choices.

As discussed in Chapter 1, theoretical studies on social interactions are related to modeling the driving forces and the consequences of the influence of social interactions, such as the herding phenomenon. Models and approaches that explicitly model agents' payoffs as a function of other agents' choices or states have appeared (Hartmann et al., 2008). In the operations management area, Veeraraghavan and Debo (2009a,b; 2010) investigate the herding behavior in queue choices when the service quality or service value is unobservable. In these studies, a long queue may be a signal of service quality in consumer decision processes (Debo et al., 2012).

The current study is also related to dynamics of a social-economic system and the marketing strategies under the influence of social interactions. To explain the phenomenon that the popular restaurant with a long queue does not increase its price, Becker (1991) develops a theoretical model where individual demand depends not only on the price but also on the demand of others due to social interactions. Brock and Durlauf (2001) investigate the rational expectations equilibria in the discrete choice model where an individual's utility depends on one's own choice (the private utility) and others' choices (the social utility). In the framework of Lopez-Pintado and Watts (2008) on modeling binary decisions with social interactions, a one-dimensional "influence-response function" in terms of the (weighted) number of others choosing each of the alternatives is developed. Ellison and Fudenberg (1995) and Banerjee and Fudenberg (2004) investigate the dynamics and equilibrium of an economic system where individuals get information and make decisions through WOM learning. Several studies focus on marketing strategies envisioning the influence of social interactions in order to increase profitability, such as the pricing strategy with network externalities (Xie and Sirbu, 1995; Bensaid and Lesne, 1996; Cabral et al., 1999; Jing et al., 2011), the optimal entry timing decision into a new market (Joshi et al., 2009), product design (Aral and Walker, 2011) and the strategy of providing online customer review (Chen and Xie, 2008). Sapra et al. (2010) consider the inventory replenishment decision when the shortages as a signal of popularity may induce more demand of a product.

Although the influence of social interactions is well recognized, few studies have focused on the link between operations and the influence of social interactions, especially when operations are subject to capacity constraint. In this chapter, we investigate the capacity and price decision under social interactions, with a focus on how operational decisions should be made to balance the tradeoff between social interaction effect and congestion externality in profit optimization is the main focus.

Tab. 3.1: Key variables and notations.

Variables	Notations	Variables	Notations
Capacity/Service rate	μ	Discount factor	δ
Price	p	Social interaction intensity	α
Arrival rate	λ	Waiting cost per unit time	w
Social value	$v^s(\lambda)$	Consumer expected surplus	S
Total potential market size	Λ	Consumer reservation surplus	S^r
Cost per capacity per customer	β	Intrinsic service value	v

3.4 Capacity and Price Decisions Under Social Interactions

This section describes the capacity and price decision model under social interactions. A monopoly service provider offers frequently purchased service to the consumer market where consumer purchase decisions are influenced by social interactions. The objective of the monopolist is to maximize the long-run discounted profit through capacity/service rate μ and service price p. The monopolist is operated through a queueing system. Due to capacity constraint and random arrivals as well as random service time, queues will be formed. Consumers need to decide whether to join the queue to purchase the service or balk. In the following section, we use the purchase decision and the queue-joining decision interchangeably. We assume the queue length is unobservable to consumers before joining the queue. Consumers are strategic in queue-joining decision as in Hassin and Haviv (2003) which is modeled in the following section. Although we assume the queue is unobservable, the model is also applicable to the situation where the system is in equilibrium in each period and we focus on the equilibrium dynamics of the service system. Key notations used in this chapter are listed in Table 3.1.

3.4.1 Consumer Queue-joining Decision

Due to the influence of social interactions, from consumer perspective, the *perceived value* V of the service depends on its *intrinsic value* v and the *social value* $v^s(\lambda^h)$, where λ^h is the sales volume of the service or the *existing customer base*. For simplicity, we assume an additive form of the perceived value as $V = v + v^s(\lambda^h)$. The model specification follows the definition of social interactions used in Brock and Durlauf (2001) as *"the utility or payoff an individual receives from a given action depends directly on the choices of others in that individual's reference group"*. We realize that for observable queues, customer purchase decision may be influenced by the queue length, which has been empirically documented in Lu et al. (2012) and Kremer and Debo (2012). In this chapter, we use past *arrival rates* to measure the existing customer base, based on the assumption that the queue is unobservable.

Given the service rate μ and the price p, an individual consumer's purchase decision depends on the *expected surplus* denoted as $S(\lambda^h, \lambda, \mu, p)$ and the *reservation surplus* S^r before making the queue-joining decision. Specifically, if the expected surplus is no less than the reservation surplus, an arriving consumer will join the queue to procure the service. The expected surplus is defined as $S(\lambda^h, \lambda, \mu, p) = v + v^s(\lambda^h) - CW(\lambda, \mu) - p$, where $CW(\lambda, \mu)$ is the waiting cost due to congestion in the queue. Since the queue is unobservable, we assume consumers adopt the expected waiting time (in queue and service) to estimate their waiting cost, and the resulting arrival rate λ is denoted as the *equilibrium arrival rate*. We assume the monopolist is operated as an M/M/1 queue for tractability (the model can be extended to general queueing models). Therefore, demand arrives according to a Poisson process and the service time is exponentially distributed. Although the M/M/1 model may not capture all aspects of the system behavior, it captures many of the congestion-related phenomenon (Ata and Shneorson, 2006). Moreover, using this model, we are able to explicitly characterize the operational policy and the structural properties of the system dynamics. Therefore, consumer expected waiting cost is calculated as

$$CW(\lambda, \mu) = \begin{cases} \frac{w}{\mu - \lambda}, & 0 \leq \lambda < \mu \\ +\infty, & \mu \leq \lambda \leq \Lambda \end{cases}$$

where w is the waiting cost per unit time for all customers, and Λ is the total potential market size (or arrival rate).

We assume consumers are *homogeneous* in terms of the reservation surplus S^r. Since the queue is unobservable, upon arrival, each consumer makes the joining decision based on the perceived value of the service, the expected waiting cost and the price. We focus on the symmetric *equilibrium* queue-joining strategy since all consumers are homogeneous as in Hassin and Haviv (2003) and Anand et al. (2011). Letting $\gamma_e(\mu, p, \lambda^h)$ denote the equilibrium probability that an individual would join the queue given the service rate μ and price p as well as the existing customer base λ^h, we have the following scenarios:

- If $S(\lambda^h, \Lambda, \mu, p) = v + v^s(\lambda^h) - CW(\Lambda, \mu) - p > S^r$, i.e., the expected surplus is larger than the reservation for an individual consumer even if all the other consumers join the queue; therefore, all consumers will join the queue in equilibrium, i.e., $\gamma_e(\mu, p, \lambda^h) = 1$;

- If $S(\lambda^h, 0, \mu, p) = v + v^s(\lambda^h) - CW(0, \mu) - p < S^r$, i.e., the expected surplus is smaller than the reservation for an individual consumer even if no consumers join the queue; therefore, no consumers will join the queue in equilibrium, i.e., $\gamma_e(\mu, p, \lambda^h) = 0$;

- If $\exists \lambda \in [0, \Lambda]$, $S(\lambda^h, \lambda, \mu, p) = v + v^s(\lambda^h) - CW(\lambda, \mu) - p = S^r$, i.e., each individual consumer plays a mixed strategy in equilibrium, in the sense that each consumer will join the queue with probability $\gamma_e(\mu, p, \lambda^h) = \frac{\lambda}{\Lambda} \in [0, 1]$.

Therefore, the equilibrium arrival rate denoted as $\lambda^e(\lambda^h, \mu, p)$ is given as the following result:

$$\lambda^e(\lambda^h, \mu, p) = \begin{cases} \Lambda, & S(\lambda^h, \Lambda, \mu, p) > S^r \\ \lambda \in [0, \Lambda], & S(\lambda^h, \lambda, \mu, p) = S^r \\ 0, & S(\lambda^h, 0, \mu, p) < S^r \end{cases} .$$

Given consumer queue-joining strategy, optimal operational decisions will drive consumer expected surplus to the reservation level, if the expected surplus is larger than the reservation; the monopolist can always make additional profit by increasing the price or decreasing the service rate to reduce consumer expected surplus to the reservation level meanwhile keeping the same demand.

3.4.2 Capacity and Price Decisions

We consider a monopolist service provider which offers frequently purchased service in an infinite-horizon with periods $t = 1, 2, \cdots, \infty$ in the consumer market with potential demand size Λ_t. If the service is not frequently consumed, we interpret Λ_t as the market size of new consumers who enter the market in period t. Given the price of the service p_t, the monopolist wants to cover the market as much as possible in each period. However, due to capacity constraint of the service rate μ_t, the market that can be covered by the monopolist is restricted due to congestion. The monopolist can increase its service rate in order to serve more consumers (or cover a larger market) at the marginal cost β per unit of service rate denoted as $C(\mu_t) = \beta\mu_t$, such as training of employees, facility improvement, hiring more employees and so on. In this chapter, we assume the monopolist can not simply increase the service speed meanwhile keeping the intrinsic service value unchanged. For example, in *consumer-intensive service* industry studied by Anand et al. (2011), such as the consulting service, simply increasing the service speed may decrease the service value. Therefore, in each period, the monopolist needs to make the price and service rate decisions.

The total potential market size Λ_t may be influenced by the price, the service rate or the existing customer base, which may be dynamically changed in each period. We model the service setting with a queueing regime with unobservable queues. Therefore, Λ_t is the total potential arrival rate, and the effective arrival rate is $\lambda_t \leq [0, \Lambda_t]$ in period t after the price p_t and service rate μ_t are determined. We assume the firm is operated with a risk neutral manager whose objective is to maximize the long-run discounted profit with *discount factor* $\delta \in (0, 1)$. We refer to the equilibrium arrival rate as the *state* in each period; while the *steady state* refers to the convergence of the equilibrium arrival rate in the dynamics of the service operations model.

Based on the above consumer purchase decision, in each period t, due to social interactions, given μ_t and p_t, consumer expected surplus is given as

$$S(\lambda^{\mathbf{h}}_{\mathbf{t-1}}, \lambda_t, \mu_t, p_t) = v + v(\lambda_{\mathbf{t-1}}) - CW(\lambda_t, \mu_t) - p_t,\ t \geq 1$$

where $\lambda^{\mathbf{h}}_{\mathbf{t-1}} = (\lambda^h_0, \lambda^h_1, \cdots, \lambda^h_{t-1})$ denotes the effective arrival rates during previous periods. Therefore, we assume consumer perceived value of the service in period t before joining the queue is influenced by the arrival rates in previous periods which serve as the existing customer base. We assume the *social value* of the service due to the influence of social interactions as $v(\lambda^{\mathbf{h}}_{\mathbf{t-1}}) = \alpha\lambda^h_{t-1}$, where $\alpha \geq 0$ is denoted as the *social interaction intensity*. The magnitude of α has been measured by various empirical studies. For example, in Hartmann (2010), 35% of the median consumer value is attributable to the influence of social interactions. Although several empirical studies do not directly measure the impact of existing customer base or sales volume on the valuation of potential customers, they have measured the effect of OL or WOM on the sales volume. For example, in the dining choice experiment (Cai et al., 2009), the sales quantity can be increased by 13 percent to 20 percent; in the online vouchers sales in Groupon.com, a 10% increase in past sales is associated with 1.4 more vouchers in the next hour (Li and Wu, 2012); displaying popularity information of the online wedding service can bring 30.53 more clicks (Tucker and Zhang, 2011).

Since in each period, the optimal service rate and the price will reduce consumer expected surplus to be equal to the reservation level, the equilibrium arrival rate in period t satisfies

$$S(\lambda^{\mathbf{h}}_{\mathbf{t-1}}, \lambda_t, \mu_t, p_t) = v + \alpha\lambda^h_{t-1} - CW(\lambda_t, \mu_t) - p_t = S^r_t,\ \lambda_t \in [0, \Lambda_t],\, t \geq 1$$

where S^r_t is consumer reservation surplus in period t. We normalize the reservation surplus as $S^r = 0$ in each period in the following sections, and the potential arrival rate Λ_t is large enough which does not impact the operational decisions. In period $t+1$, the equilibrium arrival rate λ_t in period t will be the effective arrival rate λ^h_t. Therefore, in the following section, we omit the superscript and denote the state transition function (or the arrival rate dynamics) as $\lambda_t = F(\lambda_{t-1}, \mu_t, p_t)$.

The profit of the monopolist in period t is given as

$$\pi(\mu_t, p_t, \lambda_{t-1}) = (p_t - \beta\mu_t)F(\lambda_{t-1}, \mu_t, p_t) \tag{3.4.1}$$

and the long-run discounted profit with the initial arrival rate λ_0 and the discount factor $\delta \in (0,1)$ under the policy $(\boldsymbol{\mu_t}, \mathbf{p_t})$ is formulated as

$$\Pi(\lambda_0) = \sum_{t=0}^{\infty} \delta^t \pi(\mu_{t+1}, p_{t+1}, \lambda_t),\ \ s.t.\, \lambda_t = F(\lambda_{t-1}, \mu_t, p_t),\ \ t \geq 1 \tag{3.4.2}$$

The objective of the monopolist is to maximize the long-run discounted profit through service rate and price decisions as

$$V(\lambda_0) = \sup_{\mu_t, p_t} \Pi(\lambda_0) \tag{3.4.3}$$

where the value function $V(\lambda)$ is the unique bounded solution to the following Bellman equation (Stokey et al., 1989):

$$V(\lambda) = \sup_{\mu, p} \pi(\mu, p, \lambda) + \delta V\left(F(\lambda, \mu, p)\right) \tag{3.4.4}$$

since the single period profit $\pi(\mu, p, \lambda)$ is bounded.

There are two decision variables in each period, in the following section, we first consider three special dynamical models, namely the model with constant price and service rate and the model with constant price or constant service rate. We then consider the general model, where the service rate and the price are simultaneously determined in each period.

3.5 Service Operations with Constant Price and Capacity

In this section, we consider a special strategy adopted by the monopolist, *the constant price and service rate policy*, where the service rate and the price are pre-determined as $\mu_t = \mu$ and $p_t = p \geq \beta\mu$. These fixed operational decisions capture the situation where firms announce the price and service rate before entering into the market and stick to their policies in operations. Therefore, for each served consumer, the monopolist receives a constant profit margin $p - \beta\mu$ indicating the monopolist adopts a *constant price premium* (or profit marginal) policy. We assume the following condition holds in this section:

Assumption on constant price and service rate: $v - p - \frac{w}{\mu} > 0$.

The above assumption indicates even if there is no existing customer base, i.e., $\lambda_t = 0$, consumers still have incentives to adopt the service in period $t+1$, i.e., the equilibrium arrival rate in each period is always positive (we may not need this assumption, if the initial arrival rate is positive).

The arrival rate dynamic is through the following state transition equation:

$$v + \alpha\lambda_{t-1} - p - \frac{w}{\mu - \lambda_t} = 0 \Rightarrow \lambda_t = \mu - \frac{w}{v + \alpha\lambda_{t-1} - p}, \ \lambda_t < \mu$$

Therefore, the arrival rate in period t is increasing in the arrival rate in the previous period (the existing consumer base). The *arrival rate path* $\{\lambda_t, t \geq 1\}$ starting with the initial arrival rate λ_0 can be computed sequentially given the service rate and price. If there exists a *steady state arrival rate*, denoted as λ^c, we can solve λ^c as the solution to the following quadratic equation: $\lambda = \mu - \frac{w}{v+\alpha\lambda-p}$. It is easy to see that there exists a unique solution denoted as $\lambda^c(\mu, p)$, where

$$\lambda^c = \lambda^c(\mu, p) = \frac{\alpha\mu - v + p + \sqrt{(\alpha\mu + v - p)^2 - 4\alpha w}}{2\alpha} \in (0, \mu)$$

based on the above assumption. It can be shown that $\lambda^c(\mu, p)$ is increasing in α, i.e., the larger social interaction intensity, the larger steady state arrival rate. Thus, a large social interaction intensity will help the service provider to cover a large market eventually.

We have the following result in terms of the existence of the steady state arrival rate and the monotonicity of the arrival rate paths:

Lemma 3.1. *Under the constant price and service rate policy, there exists a unique steady state arrival rate λ^c and the arrival rate paths $\{\lambda_t, t \geq 1\}$ converge to λ^c monotonically.*

Proof. Since λ_t is increasing in λ_{t-1}, and bounded above by μ, there exists a unique steady state λ^c in $(0, \mu)$. If $\lambda_0 \geq \lambda^c$, the sequence converges to λ^c decreasingly, while if $\lambda_0 < \lambda^c$, the sequence converges to λ^c increasingly. □

Remark 3.1. The arrival rate is bounded in $\lambda_t \in \Lambda = [\min(\lambda_0, \lambda^c), \max(\lambda_0, \lambda^c)]$. The steady state arrival rate is decreasing and concave in p, while increasing and concave in μ, thus denoted as $\lambda^c(\mu, p)$. It is also supermodular, since the cross derivative denoted as $\lambda_{\mu p}$ can be checked to be positive.

Given the constant service rate μ and price p, the single period profit is reformulated in terms of the arrival rate as $\pi(\lambda_{t-1}, \lambda_t) = (p - \beta\mu)\lambda_t, \ \lambda_t = \left(\mu - \frac{w}{v-p+\alpha\lambda_{t-1}}\right)$, which is increasing in the state λ_{t-1}. Based on the above result, if $\lambda_0 \geq \lambda^c$, the profit function will decrease monotonically, i.e., $\pi(\lambda_{t-1}, \lambda_t) \geq \pi(\lambda_t, \lambda_{t+1})$; while if $\lambda_0 < \lambda^c$, the profit function will increase monotonically, i.e., $\pi(\lambda_{t-1}, \lambda_t) < \pi(\lambda_t, \lambda_{t+1})$. For the fixed (μ, p) policy, given the initial arrival rate $\lambda = \lambda_0$, the long-run discounted

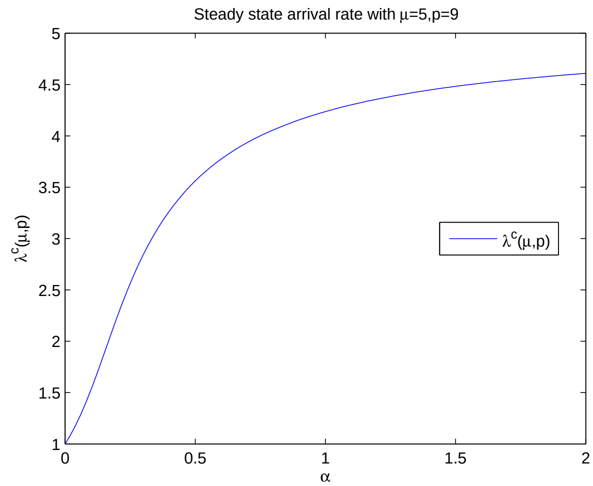

Fig. 3.5.1: The steady state arrival rate with respect to the social interaction intensity $\alpha \in [0, 2]$ under the constant service rate and price policy.

profit is denoted as

$$W(\mu, p, \lambda_0) = \sum_{t=0}^{\infty} \delta^t (p - \beta\mu)\lambda_{t+1}, \;\; \lambda_{t+1} = \left(\mu - \frac{w}{v - p + \alpha\lambda_t}\right) \tag{3.5.1}$$

which is bounded, since the single period profit is bounded.

Fig.3.5.1-3.5.3 describe the steady state arrival rate with respect to α, two arrival rate paths and two profit paths with $\alpha = 0.4$ and $\alpha = 0.6$ under the parameters $v = 10, \;\beta = 1, \;w = 1$ and $\mu = 5, \;p = 9$.

The optimal constant service rate and price policy denoted as $(\mu^\star, p^\star)$ is to maximize the long-run discounted profit $W(\mu, p, \lambda_0)$, as

$$V(\lambda_0) = \max_{\mu,p} W(\mu, p, \lambda_0) \tag{3.5.2}$$

Although given μ and p, the profit in each period is monotonic, due to the nonlinear state transition function; to solve the optimal constant service rate and price is not straightforward. In the following section, we focus on two special policies, namely, the policy that maximizes the steady state profit and the policy that maximizes the first period profit. We focus on the optimal constant service rate and price from the above two special policies. We define

$$\max_{\mu,p} \pi(\mu, p, \lambda) \equiv \max_{\mu} \max_{p} \pi(\mu, p, \lambda) \tag{3.5.3}$$

and we denote the optimal decisions as $(\mu, p(\mu))$. The above definition implies the

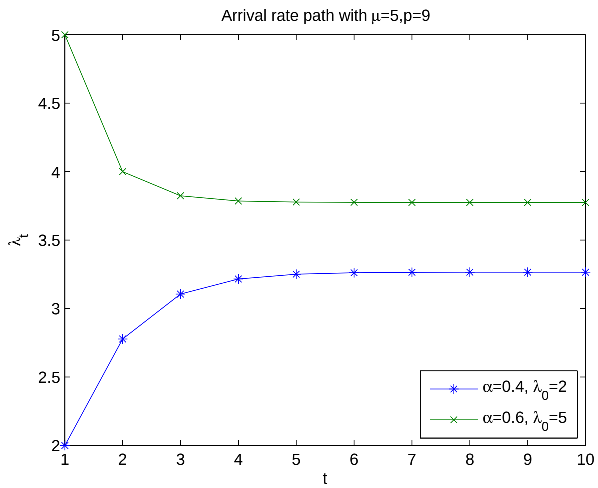

Fig. 3.5.2: Two arrival rate paths under the constant service rate and price policy.

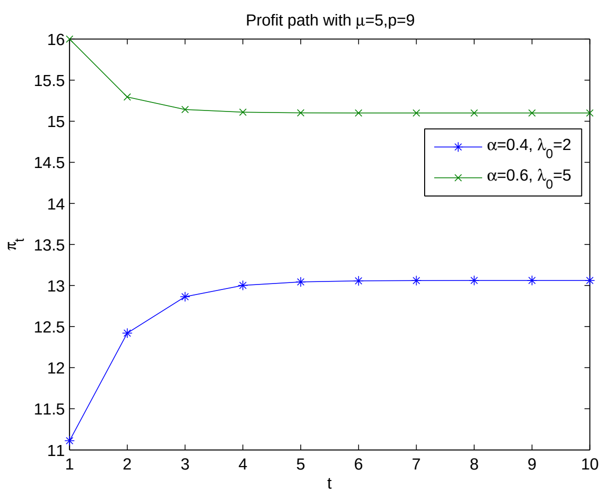

Fig. 3.5.3: Two profit paths under the constant service rate and price policy.

monopolist sequentially solves the service rate μ and price p. Since the profit function $\pi(\mu, p, \lambda)$ is concave in p and μ respectively and supermodular in (μ, p), there exists a unique pair of $(\mu, p(\mu))$ for fixed λ.

The profit in the first period is

$$\pi_1(\mu, p, \lambda_0) = (p - \beta\mu)\lambda_1 = (p - \beta\mu)\left(\mu - \frac{w}{v + \alpha\lambda_0 - p}\right) \tag{3.5.4}$$

and the optimal decision is denoted as $\left(\mu^I, p(\mu^I)\right)$ which maximizes $\pi_1(\mu, p, \lambda_0)$. The corresponding steady state arrival rate is denoted as $\lambda^c\left(\mu^I, p(\mu^I)\right)$ and the optimal profit in the first period is denoted as $\pi_1\left(\mu^I, p(\mu^I)\right)$.

The profit in the steady state is

$$\pi^c(\mu, p, \lambda^c) = (p - \beta\mu)\left(\frac{\alpha\mu - v + p + \sqrt{(\alpha\mu + v - p)^2 - 4\alpha w}}{2\alpha}\right) \tag{3.5.5}$$

and the unique optimal service rate and price is denoted as $\left(\mu^{II}, p(\mu^{II})\right)$ that maximizes the steady state profit. The corresponding steady state arrival rate is denoted as $\lambda^c\left(\mu^{II}, p(\mu^{II})\right)$ and the optimal profit in steady state is denoted as $\pi^c\left(\mu^{II}, p(\mu^{II})\right)$.

Based on the two policies, denote $\lambda_{\min} = \min\left(\lambda^c\left(\mu^I, p(\mu^I)\right), \lambda^c\left(\mu^{II}, p(\mu^{II})\right)\right)$, and $\lambda_{\max} = \max\left(\lambda^c\left(\mu^I, p(\mu^I)\right), \lambda^c\left(\mu^{II}, p(\mu^{II})\right)\right)$ and the long-run discounted profit with the policy $\left(\mu^I, p(\mu^I)\right)$ and $\left(\mu^{II}, p(\mu^{II})\right)$ as $W^I(\lambda_0)$ and $W^{II}(\lambda_0)$ respectively, and $W(\lambda_0) = \max\left(W^I(\lambda_0), W^{II}(\lambda_0)\right)$ as the optimal long-run discounted profit based on the two constant policies. Therefore, given the two feasible policies, we have the following results:

Proposition 3.1. *Given the initial arrival rate λ_0 and the two feasible policies $(\mu^I, p(\mu^I))$ and $(\mu^{II}, p(\mu^{II}))$, the following cases hold:*

- *if $\lambda_0 \leq \lambda_{\min}$, $W(\lambda_0) \leq \frac{\pi^c\left(\mu^{II}, p(\mu^{II})\right)}{1-\delta}$;*
- *if $\lambda_0 \geq \lambda_{\max}$, $W(\lambda_0) \leq \frac{\pi_1\left(\mu^I, p(\mu^I)\right)}{1-\delta}$;*
- *if $\lambda_{\min} < \lambda_0 < \lambda_{\max}$, $W(\lambda_0) \leq \max\left(\frac{\pi^c\left(\mu^{II}, p(\mu^{II})\right)}{1-\delta}, \frac{\pi_1\left(\mu^I, p(\mu^I)\right)}{1-\delta}\right)$;*

Given the constant service rate and price policy, the impact of social interactions on the dynamics of the service system depends on the comparison between the steady state arrival rate and the initial arrival rate. Based on the previous result, given the initial arrival rate λ_0, a large social interaction intensity α will induce a large λ^c, which indicates the profit path under either policy may increase; while a small social

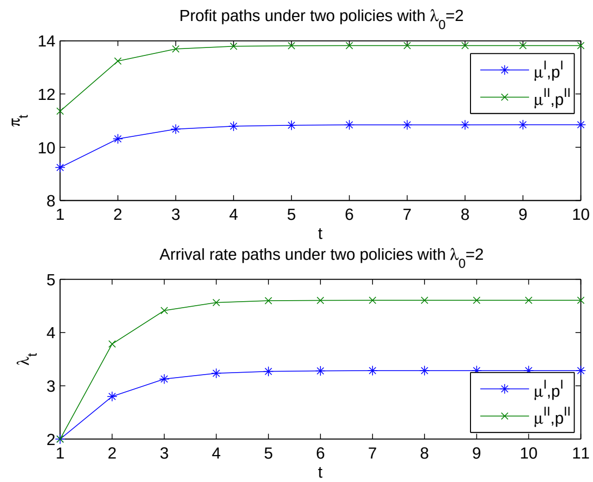

Fig. 3.5.4: Profit paths and arrival rate paths under the two constant service rate and price policies.

interaction intensity α will lead to a small steady state arrival rate, which indicates the profit path will decrease; while a medium social interaction intensity α may lead to a decreasing profit path under $(\mu^I, p(\mu^I))$, with an increasing profit path under $(\mu^{II}, p(\mu^{II}))$. Therefore, if the social interaction intensity is large, it will be optimal for the monopolist to adopt the policy $(\mu^{II}, p(\mu^{II}))$; while for a small social interaction intensity, the policy $(\mu^I, p(\mu^I))$ may be optimal.

Based on the parameters $v = 10,\ \beta = 1,\ w = 4,\ \alpha = 0.3$, the optimal service rate and price that maximize the steady state profit are $\mu^{II} = 6.2856,\ p^{II} = 9.2857$. Fig.3.5.4-3.5.6 depict the profit paths under the two policies with the initial arrival rates $\lambda_0 = 2$, $\lambda_0 = 5$ and $\lambda_0 = 4$ in correspondence with the above three cases. The optimal service rate and price which maximize the first period profit with $\lambda_0 = 4$ are $\mu^I = 5.6001,\ p^I = 9.2001$, and $\mu^I = 5.3000,\ p^I = 8.6001$ with $\lambda_0 = 2$, while $\mu^I = 5.5954,\ p^I = 9.2851$ with $\lambda_0 = 5$.

3.6 *Dynamic Capacity Management Under Social Interactions*

3.6.1 *Model Settings*

In this section, we consider the dynamic capacity management under the influence of social interactions, where the price is fixed as a constant due to regulations or other factors, such as the price promise of the service provider. While we assume the service rate can be adjusted by the monopolist in each period, the monopolist decides the optimal service rate in order to maximize the long-run discounted profit. Since the price is fixed, we assume the following condition holds in this section:

Assumption under the constant price constraint: $Ap - \beta w \geq 0$, *where* $A =$

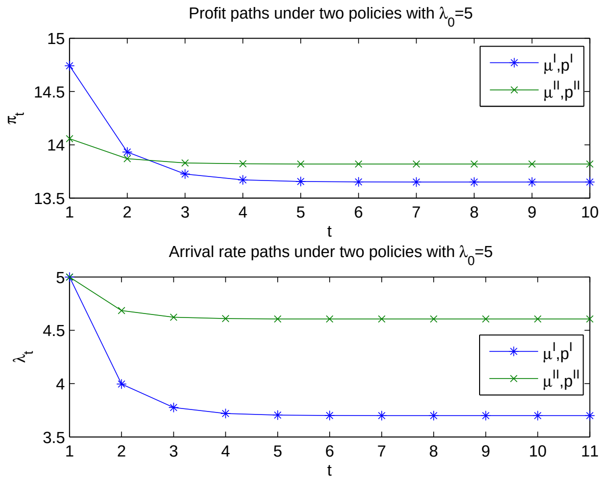

Fig. 3.5.5: Profit paths and arrival rate paths under the two constant service rate and price policies.

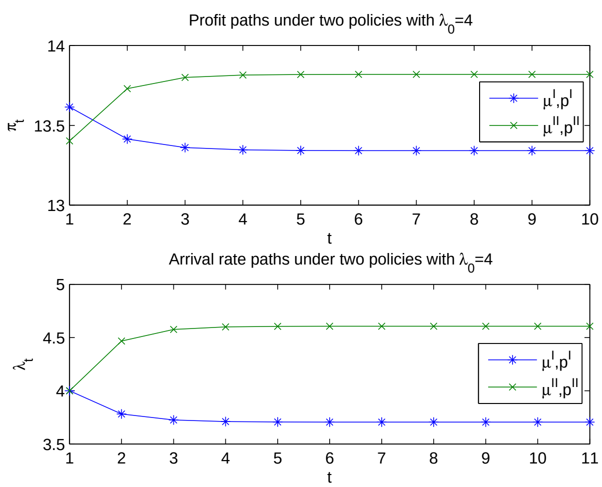

Fig. 3.5.6: Profit paths and arrival rate paths under the two constant service rate and price policies.

$v - p > 0$ *is the net value of the service from the consumer perspective.*

The assumption indicates under the fixed price p, if the monopolist sets the service rate at the maximum value $\frac{p}{\beta}$, consumers have incentive to join the queue to purchase the service even if the existing customer base is zero (hence no social interaction effect), since we have $Ap - \beta w \geq 0 \Rightarrow v - p - \frac{w}{\frac{p}{\beta}} = A - \frac{\beta w}{p} \geq 0$.

Since in equilibrium consumer reservation surplus is zero in each period, we have the following state transition function in period t:

$$v + \alpha\lambda_{t-1} - p - CW(\lambda_t, \mu_t) = 0 \Rightarrow \lambda_t = \mu_t - \frac{w}{v + \alpha\lambda_{t-1} - p} = \mu_t - \frac{w}{A + \alpha\lambda_{t-1}}$$

The service rate μ_t is bounded in the set $\mu_t \in \Omega_t = \left[\frac{w}{A+\alpha\lambda_{t-1}}, \frac{p}{\beta}\right]$, and the equilibrium arrival rate is in the interval $0 \leq \lambda_t < \mu_t$. Given the service rate, the profit in each period $t+1$ is formulated as

$$\pi(\mu_{t+1}, \lambda_t) = (p - \beta\mu_{t+1})\lambda_{t+1} = (p - \beta\mu_{t+1})\left(\mu_{t+1} - \frac{w}{A + \alpha\lambda_t}\right) \tag{3.6.1}$$

For the fixed price p, the profit is definitely bounded, since it is concave in μ_{t+1}. For an initial arrival rate λ_0, the monopolist determines the service rate μ_t in each period to maximize the long-run discounted profit as

$$V(\lambda_0) = \sup_{\mu_{t+1} \in \Omega_{t+1}} \sum_{t=0}^{\infty} \delta^t \pi(\mu_{t+1}, \lambda_t), \ s.t.\ \lambda_{t+1} = \mu_{t+1} - \frac{w}{A + \alpha\lambda_t}, \ t \geq 0 \tag{3.6.2}$$

where $\delta \in (0, 1)$ is the discount factor. The above value function is the unique bounded solution to the following Bellman equation:

$$V(\lambda) = \sup_{\mu \in \Omega} W(\lambda, \mu), \ \text{where } W(\lambda, \mu) = \pi(\mu, \lambda) + \delta V\left(\mu - \frac{w}{A + \alpha\lambda}\right) \tag{3.6.3}$$

In the above formulation, the state transition mechanism is a nonlinear function, which may cause difficulty in determining the property of the optimal service rate policy. Based on the assumption that the potential arrival rate in each period is large enough, and consumer equilibrium surplus is zero, we have

$$\mu_{t+1} = \lambda_{t+1} + \frac{w}{A + \alpha\lambda_t}$$

which is decreasing in λ_t. The above equation indicates that monopolist can determine the arrival rate (or the market coverage) in each period by setting the corresponding service rate. Therefore, we can change the decision variable from the service rate to the selection of the equilibrium arrival rate in each period. We reformulate the single period profit function as the following:

$$\pi(\mu_{t+1}, \lambda_t) = \left(p - \beta\left(\lambda_{t+1} + \frac{w}{A + \alpha\lambda_t}\right)\right)\lambda_{t+1} = \pi(\lambda_t, \lambda_{t+1}) \tag{3.6.4}$$

Obviously, the set of feasible λ_{t+1}, denoted as $\hat{\Lambda}_{t+1} = \left[0, \frac{p}{\beta} - \frac{w}{A+\alpha\lambda_t}\right]$ is bounded. Therefore, we reformulate the above problem as the following form:

$$V(\lambda_0) = \sup_{\lambda_{t+1} \in \hat{\Lambda}_{t+1}} \sum_{t=0}^{\infty} \delta^t \pi(\lambda_t, \lambda_{t+1}) \tag{3.6.5}$$

where the value function is the unique solution to the following Bellman equation

$$V(\lambda) = \sup_{\hat{\lambda} \in \hat{\Lambda}} \pi(\lambda, \hat{\lambda}) + \delta V(\hat{\lambda}). \tag{3.6.6}$$

Therefore, we have changed the original problem where the decision variable is the service rate in each period as an alternative problem where the decision variable is the equilibrium arrival rate in each period. In the following section, we will use the reformulated problem to investigate the optimal decisions in the dynamic model. The single period profit function $\pi(\lambda, \hat{\lambda})$ is increasing and concave in the state variable λ by checking the first and second order derivatives with respect to λ. Therefore, the value function $V(\lambda)$ is increasing in λ, indicating a large initial market is always beneficial to the monopolist.

Because the action set $\hat{\Lambda}$ is compact and all functions are continuous, there exists an optimal stationary arrival rate policy that solves the above problem. We define the optimal arrival rate policy as $\hat{\lambda}^\star(\lambda) = \arg\max_{\hat{\lambda}} \pi(\lambda, \hat{\lambda}) + \delta V(\hat{\lambda})$. The arrival rate path $\{\hat{\lambda}^\star_t, t \geq 1\}$ starting with the initial arrival rate λ_0 is computed sequentially as $\hat{\lambda}^\star_t = \hat{\lambda}^\star(\hat{\lambda}^\star_{t-1})$. The sequence $\{\hat{\lambda}^\star_t\}$ of the optimal decision is also the optimal state path $\{\lambda^\star_t, t \geq 1\}$, and the corresponding service rate path $\{\mu^\star_t, t \geq 1\}$ can be computed correspondingly through $\mu^\star_t = \lambda^\star_t + \frac{w}{A+\alpha\lambda^\star_{t-1}}$ in each period.

3.6.2 Myopic Policy

The monopolist that ignores the impact of the social interactions on the equilibrium arrival rate in the future period will try to maximize its profit in each period by selecting the corresponding arrival rate through the service rate decision. We define the policy that maximizes each single period profit as the *myopic policy*, since it ignores the impact of the current period decision in the future period, while we define the optimal policy as the *strategic policy*. In the following section, we use superscript M to denote the decisions and results for the myopic policy. It is easy to check that the single period profit function $\pi(\lambda, \hat{\lambda})$ is supermodular in $(\lambda, \hat{\lambda})$, based on the cross-derivative as $\pi_{\hat{\lambda}\lambda} = \frac{\alpha\beta w}{(A+\alpha\lambda)^2} > 0$. Therefore, we have the following result:

Lemma 3.2. *The single period profit maximizing arrival rate* $\tilde{\lambda}(\lambda) = \arg\max \pi(\lambda, \hat{\lambda})$ *is increasing in* λ*; while the corresponding service rate* $\tilde{\mu}(\lambda)$ *is decreasing in* λ*. Moreover, the optimal single period profit* $\pi(\lambda, \hat{\lambda})$ *is also increasing in* λ*.*

For the myopic policy, the monopolist selects the arrival rate by setting the corresponding service rate which is only optimal in the current period. The decision under the myopic policy is equivalent to setting the discount factor $\delta = 0$, in the sense that the monopolist only focuses on the current period profit and does not care about the future profit. We define the arrival rate path of the myopic policy as $\{\lambda_t^M, t \geq 1\}$ and the corresponding service rate path as $\{\mu_t^M, t \geq 1\}$. Therefore, the evolutions of the arrival rate and the corresponding service rate in the myopic policy are given as the following dynamics:

$$\lambda_t^M = \frac{p}{2\beta} - \frac{w}{2(A + \alpha\lambda_{t-1}^M)}, \; \mu_t^M = \frac{p}{2\beta} + \frac{w}{2(A + \alpha\lambda_{t-1}^M)}.$$

Since the arrival rate space is bounded and based on the monotonic property of the arrival rate dynamics, there exists a unique steady state arrival rate in the myopic policy, denoted as $\lambda^M \in (0, \mu)$, which can be solved from the equation $\lambda = \frac{p}{2\beta} - \frac{w}{2(A+\alpha\lambda)}$ as

$$\lambda^M = \frac{-(2A\beta - \alpha p) + \sqrt{(2A\beta + \alpha p)^2 - 8\alpha\beta^2 w}}{4\alpha\beta}$$

and the corresponding steady state service rate as $\mu^M = \frac{p}{2\beta} + \frac{w}{2(A+\alpha\lambda^M)}$.

Therefore, we have the monotonicity of the arrival rate path and service rate path in the myopic policy as the following result:

Lemma 3.3. *For any initial market size* λ_0*, the service rate paths* $\{\mu_t^M\}$ *and the arrival rate paths* $\{\lambda_t^M\}$ *monotonically converge to the unique steady state* μ^M *and* λ^M *respectively in opposite directions in the myopic policy.*

Proof. If $\lambda_{t-1}^M > \lambda^M$, we have $\lambda_t^M = \tilde{\lambda}(\lambda_{t-1}^M) > \tilde{\lambda}(\lambda^M) = \lambda^M$ and $\lambda_t^M - \lambda_{t-1}^M = \frac{p}{2\beta} - \frac{w}{2(A+\alpha\lambda_{t-1}^M)} - \lambda_{t-1}^M < 0$ indicating $\lambda^M < \lambda_t^M < \lambda_{t-1}^M$, i.e., the arrival rate paths $\{\lambda_t^M\}$ converge to λ^M in a decreasing order. We have $\mu_{t+1}^M = \tilde{\mu}(\lambda_t^M) > \tilde{\mu}(\lambda_{t-1}^M) = \mu_t^M$

due to the decreasing property of μ_t^M. Therefore, the service rate paths $\{\mu_t^M\}$ converge to μ^M in an increasing order. The case for $\lambda_{t-1}^M < \lambda^M$ can be similarly proved. $\square$

The above result indicates, if the initial market size is large enough, such that $\lambda_0 > \lambda^M$, in the myopic policy, the service rate path will increase gradually, while the arrival rates will decrease monotonically. However, when the initial market size is small, such that $\lambda_0 < \lambda^M$, the service rate path will decrease gradually, while the arrival rates will increase monotonically. Therefore, for a large initial market size, the myopic policy will always increase the service rate gradually; while for a small initial market size, the myopic policy will decrease the service rate gradually.

Since the myopic policy ignores the impact of the arrival rate in the current period on the future demand, we anticipate that the myopic policy selects a smaller arrival rate compared with the strategic policy. Indeed, we have the following result:

Proposition 3.2. *In the myopic policy, the arrival rate is always less than that in the strategic policy, i.e.,* $\tilde{\lambda}(\lambda) \leq \hat{\lambda}^\star(\lambda)$*, and the service rate also satisfies* $\tilde{\mu}(\lambda) \leq \hat{\mu}^\star(\lambda)$*, where* $\hat{\mu}^\star(\lambda)$ *is the corresponding service rate in the strategic policy. Moreover, for any initial arrival rate* λ_0*, the arrival rate in the myopic policy is always less than that in the strategic policy, i.e.,* $\lambda_t^M \leq \lambda_t^\star$.

Proof. The value function $V(\lambda)$ is increasing in the state λ, indicating

$$\hat{\lambda}^\star(\lambda) = \arg\max_{\hat{\lambda}\in\hat{\Lambda}} \left\{\pi(\lambda,\hat{\lambda}) + \delta V(\hat{\lambda})\right\} \geq \arg\max_{\hat{\lambda}\in\hat{\Lambda}} \pi(\lambda,\hat{\lambda}) = \tilde{\lambda}(\lambda)$$

We also have $\hat{\mu}^\star(\lambda) = \hat{\lambda}^\star(\lambda) + \frac{w}{A+\alpha\lambda} \geq \tilde{\lambda}(\lambda) + \frac{w}{A+\alpha\lambda} = \tilde{\mu}(\lambda)$. We use induction to prove the rest. For any initial arrival rate λ_0, we have $\lambda_1^\star = \hat{\lambda}^\star(\lambda_0) \geq \tilde{\lambda}(\lambda_0) = \lambda_1^M$. Suppose $\lambda_t^\star \geq \lambda_t^M$. We have $\lambda_{t+1}^\star = \hat{\lambda}^\star(\lambda_t^\star) \geq \tilde{\lambda}(\lambda_t^\star) \geq \tilde{\lambda}(\lambda_t^M) = \lambda_{t+1}^M$ based on the increasing property of $\tilde{\lambda}(\lambda)$. Therefore, $\lambda_t^\star \geq \lambda_t^M$ for any $t \geq 1$. $\square$

Therefore, in the myopic policy, the arrival rate path is always less than the optimal arrival rate path in the strategic policy for the same initial arrival rate λ_0. The arrival rates in the myopic policy obviously lower consumer perceived value of the service in the future period, and thus erode the future profit due to a smaller effective arrival rate. By selecting a higher arrival rate through a larger service rate decision, the strategic policy trades off the current period profit for future period benefit under the influence of social interactions. In order to utilize the influence of social interactions for the long term operations provides an alternative reason to explain why some companies adopt seemingly suboptimal operational decisions in the short term situation, such as a higher service delivery speed (as one aspect of service quality) or a higher capacity.

3.6.3 Strategic Policy

Since there exists a unique steady state under the myopic policy for both the arrival rate and the service rate, we investigate whether the steady state in the strategic policy exists or not. Suppose there exists a steady state arrival rate $\lambda^{\star\star}$ and correspondingly the service rate is $\mu^{\star\star}$. If the monopolist sets the service rate at $\mu^{\star\star}$ when the existing arrival rate is $\lambda^{\star\star}$, the effective arrival rate will be the same as $\lambda^{\star\star} = \mu^{\star\star} - \frac{w}{A+\alpha\lambda^{\star\star}}$ in the current period. Therefore, if the monopolist has an initial market size $\lambda^{\star\star}$, the optimal arrival rate chosen each period will be $\lambda^{\star\star} = \arg\max_{\hat{\lambda}\in\hat{\Lambda}} \left\{\pi(\lambda^{\star\star}, \hat{\lambda}) + \delta V(\hat{\lambda})\right\}$, and the long-run discounted profit is calculated as $V(\lambda^{\star\star}) = \sum_{t=0}^{\infty} \delta^t \pi(\lambda^{\star\star}, \lambda^{\star\star}) = \frac{\pi(\lambda^{\star\star},\lambda^{\star\star})}{1-\delta}$.

In the following section, we first prove the existence and the uniqueness of the steady state in the strategic policy and then characterize the solution of the steady state arrival rate and service rate. We first have the monotonicity of the optimal arrival rate path in the strategic policy:

Lemma 3.4. *The optimal arrival rate $\hat{\lambda}^\star(\lambda)$ in the strategic policy is increasing in λ. Moreover, the optimal arrival rate path $\{\lambda_t^\star, t \geq 1\}$ is monotonic.*

The monotonicity of the strategic policy can be seen from the following example. Given the initial arrival rate $\lambda_0' > \lambda_0$. Define the optimal arrival rate path as $\{\lambda_0^\star, \lambda_1^\star, \lambda_2^\star, \cdots\}$ for $\lambda_0 = \lambda_0^\star$. We have $V(\lambda_0) = \sum_{t=0}^{\infty} \delta^t \pi(\lambda_t^\star, \lambda_{t+1}^\star)$. We consider a feasible arrival rate path for λ_0', such that $\{\lambda_0', \lambda_1', \lambda_2^\star, \cdots\}$ where $\lambda_1' > \lambda_1^\star$ and $\pi(\lambda_0, \lambda_1^\star) = \pi(\lambda_0', \lambda_1')$. Such a λ_1' must exist, since $\pi(\lambda, \hat{\lambda})$ is increasing in the state variable λ, and if $\hat{\lambda} \geq \hat{\lambda}^\star(\lambda)$, $\pi(\lambda, \hat{\lambda})$ is decreasing in $\hat{\lambda}$, we have $\pi(\lambda_0, \lambda_1^\star) < \pi(\lambda_0', \lambda_1^\star) > \pi(\lambda_0', \lambda_1')$. Therefore, we have $\pi(\lambda_1', \lambda_2^\star) > \pi(\lambda_1^\star, \lambda_2^\star)$ and

$$V(\lambda_0') \geq \pi(\lambda_0', \lambda_1') + \delta\pi(\lambda_1', \lambda_2^\star) + \sum_{t=2}^{\infty} \delta^t \pi(\lambda_t^\star, \lambda_{t+1}^\star) > V(\lambda_0)$$

which indicates that starting with a larger initial market size λ_0', the monopolist can select a larger arrival rate in period 1 and then selects the arrival rate path in the following periods as the optimal one for the smaller initial market, and the profit is always larger. We continue this reasoning by selecting a larger arrival rate in period 2, 3, ... , and eventually we get the optimal arrival rate path for the larger initial market. Therefore, the optimal arrival rate $\hat{\lambda}^\star(\lambda)$ is always increasing in λ.

Although the optimal arrival rate path is monotonic, the corresponding service rate path may not be monotonic, since $\mu^{\star}_{t+1}$ is increasing in $\lambda^{\star}_{t+1}$ while decreasing in λ_t. On the one hand, in order to cover a larger arrival rate, the service rate $\mu^{\star}_{t+1}$ will become larger; on the other hand, a large existing customer base λ_t will motivate the firm to offer a slower service rate, since the service rate is costly. The impacts of these two forces depend on the parameters α, w, A.

Based on the above monotonicity of the optimal arrival rate, we have the following result in terms of the existence and the uniqueness of the steady state in the strategic policy:

Proposition 3.3. *There exists a unique steady state service rate $\mu^{\star\star}$ and arrival rate $\lambda^{\star\star}$ in the strategic policy. All arrival rate paths converge to the steady state $\lambda^{\star\star}$ monotonically.*

Proof. Since the arrival rate is bounded and the optimal arrival rate path is monotonic, we conclude the optimal arrival rate path will converge to a unique steady state $\lambda^{\star\star}$. The corresponding service rate will converge to the unique steady state $\mu^{\star\star}$. Since the arrival rate path is monotonic, all arrival rate paths converge to the steady state $\lambda^{\star\star}$ monotonically. □

Therefore, starting with a larger initial market, the monopolist will systematically lower the arrival rate, which eventually converges to the unique steady state. While starting with a smaller initial market, the optimal arrival rate will increase gradually and eventually converge to the steady state. Although a large customer base or market size will be preferred by companies, due to capacity constraint, firms have to reduce the demand since capacity is costly. This may explain why some companies with popular products or services have to maintain certain waiting lists and keep a long queue.

We characterize the steady state arrival rate and the service rate in the strategic policy, as shown in the following result:

Theorem 3.1. *In the strategic policy, the steady state arrival rate $\lambda^{\star\star}$ is the unique solution in $(0, \frac{p}{\beta})$ to the cubic function $(p-2\beta\lambda)(A+\alpha\lambda)^2-\beta w(A+\alpha\lambda)+\delta\alpha\beta w\lambda = 0$. The steady state service rate is $\mu^{\star\star} = \lambda^{\star\star} + \frac{w}{A+\alpha\lambda^{\star\star}}$. The steady state $\lambda^{\star\star}$ in the strategic policy is increasing in α and δ respectively. Moreover, $\lambda^{\star\star}$ is larger than the steady state in the myopic policy λ^M, i.e., $\lambda^{\star\star} \geq \lambda^M$.*

Therefore, the steady state arrival rate in the strategic policy can be easily solved from a cubic function. If the magnitude of the social interaction intensity is larger, the steady state arrival rate in the strategic policy will also be larger. Thus, starting from the same initial arrival rate λ_0, if $\alpha_1 \geq \alpha_2$, we have the corresponding relation $\lambda_t^\star(\alpha_1) \geq \lambda_t^\star(\alpha_2)$, i.e., for any point in time, the arrival rate path with a larger social interaction intensity will be always larger than that with a smaller social interaction intensity, if the monopolist adopts the strategic policy. We also know that the optimal arrival rate in the strategic policy is larger than that in the myopic policy. Therefore, a large social interaction intensity also indicates the monopolist should trade off more current period profit to increase future period demand and profit. Similarly, if the discount factor is bigger, the steady state in the strategic policy will also be larger, indicating that starting from the same initial arrival rate λ_0, if $\delta_1 \geq \delta_2$, we have the corresponding relation $\lambda_t^\star(\delta_1) \geq \lambda_t^\star(\delta_2)$. Therefore, the strategic policy will trade off more current profit for the long term benefit in the future period, if the monopolist cares about more future profit. The above result also provides guidance for a profit maximizing firm in capacity decision-making. For example, managers can invest in the capacity at the steady state level $\mu^{\star\star}$, so that eventually the arrival rate will converge to the steady state, no matter how large the initial market is. If the arrival rate can converge to the steady state very fast, it would be an appropriate capacity strategy. Intuitively, a higher price level will lead to a smaller steady state arrival rate, which can be seen from the dynamics of the system.

Based on the M/M/1 queue, the queue length is calculated as $L = \frac{\lambda}{\mu-\lambda} = \frac{\rho}{1-\rho}$, where $\rho = \frac{\lambda}{\mu}$ is the utilization. From the above result, we can easily see that due to a larger arrival rate, the queue length and the expected waiting time in steady state under the strategic policy are also larger than those under the myopic police. The steady state utilization in the strategic policy is also larger than that in the myopic policy.

Fig.3.6.1-3.6.4 depict the value function, the arrival rate path and the service rate path, as well as the arrival rate decision in the strategic policy and the myopic policy, with the parameters $v = 10,\ p = 9,\ \beta = 1,\ w = 1,\ \alpha = 0.8,\ \delta = 0.9$.

From the comparison in the figures, we can see, the arrival rate path and the service rate path in the strategic policy are always above those in the myopic policy. For the same customer base, the arrival rate decision in the strategic policy is always larger than that in the myopic policy. In the above result, the service rate paths in the two policies are also monotonically converging to the corresponding unique steady state.

Although the steady state arrival rate in the strategic policy is always larger than that in the myopic policy, the steady state service rate in the strategic policy may not be necessarily larger. We have the following result in terms of the steady state service rate comparison:

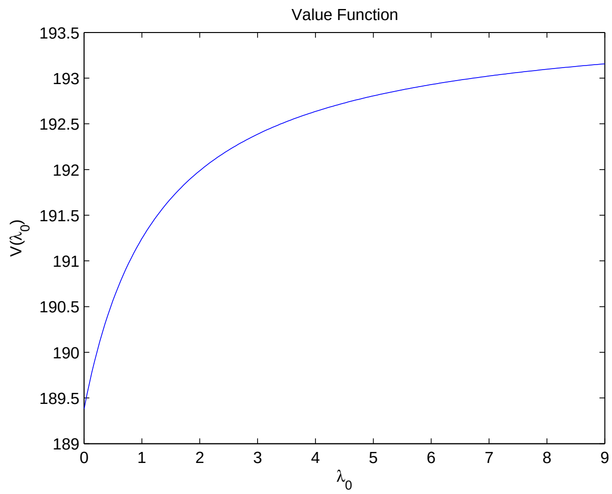

Fig. 3.6.1: Value function in the dynamic service rate management model with respect to the initial arrival rate.

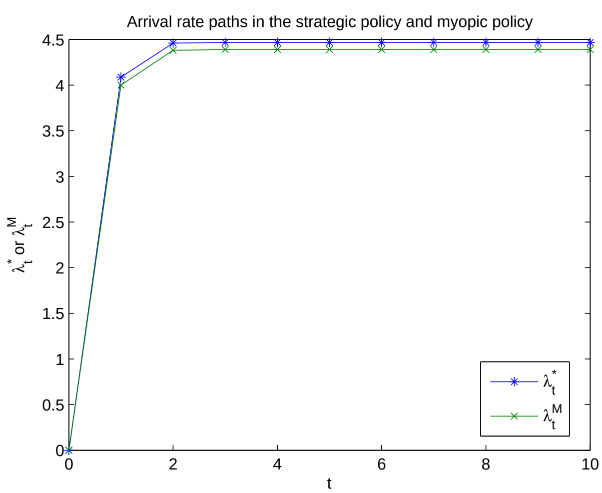

Fig. 3.6.2: Arrival rate paths in the dynamic service rate management model under the myopic policy and the strategic policy.

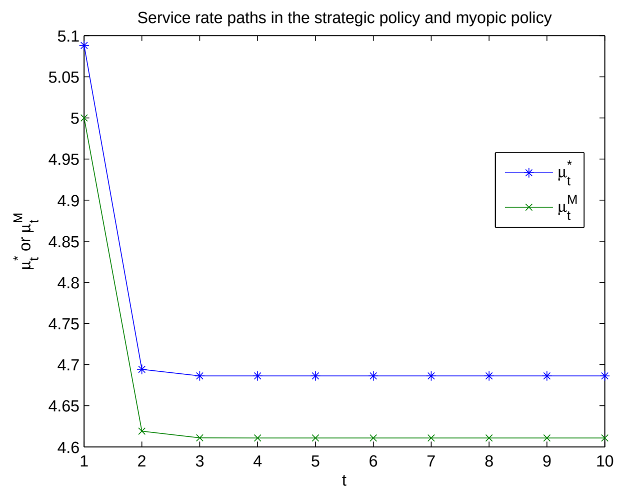

Fig. 3.6.3: Service rate paths in the dynamic service rate management model under the myopic policy and the strategic policy

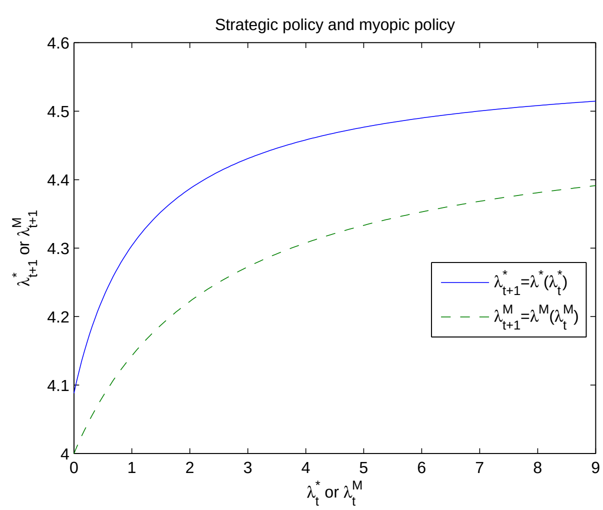

Fig. 3.6.4: Arrival rate dynamics in the dynamic service rate management model under the myopic policy and the strategic policy.

Corollary 3.1. *If* $(A + \alpha\lambda^M)^2 \geq \alpha w$*,* $\mu^{\star\star} > \mu^M$*. Otherwise, if* $\lambda^M \geq \frac{A-\sqrt{\alpha w}}{\alpha}$*,* $\mu^{\star\star} > \mu^M$*; while if* $\lambda^{\star\star} \leq \frac{A-\sqrt{\alpha w}}{\alpha}$*,* $\mu^{\star\star} < \mu^M$*.*

Proof. In both policies, the steady state service rate is $\mu = \lambda + \frac{w}{A+\alpha\lambda}$ with the first order derivative as $\frac{\partial\mu}{\partial\lambda} = 1 - \frac{\alpha w}{(A+\alpha\lambda)^2}$ which is increasing. Thus, if $(A+\alpha\lambda^M)^2 \geq \alpha w$, $\frac{\partial\mu}{\partial\lambda} = 1 - \frac{\alpha w}{(A+\alpha\lambda)^2} \geq 0$, indicating $\mu^{\star\star} > \mu^M$, since $\lambda^{\star\star} \geq \lambda^M$. Otherwise, μ is convex in λ, with the minimizer at $\lambda = \frac{A-\sqrt{\alpha w}}{\alpha}$. Thus the rest of the above result follows. $\square$

Therefore, from the above result, we can see if the steady state arrival rate is small, the service rate in the strategic policy may be smaller than that in the myopic policy. The reason is that since the strategic policy has built up a large customer base, even if the steady state service rate is smaller, the steady state arrival rate is still larger than that in the myopic policy. This result may explain why some companies can cover a larger market than their competitors, but the service delivery speed or service quality may not be superior.

In order to bound the steady state arrival rate in the strategic policy, we consider the optimal arrival rate under the constant price policy in the static single period model denoted as $\lambda^\star$, where the equilibrium arrival rate satisfies $v + \alpha\lambda - p - CW(\lambda, \mu) = 0 \Rightarrow \mu = \lambda + \frac{w}{A+\alpha\lambda}$ with the profit function

$$\pi(\lambda, \lambda) = \left(p - \beta\left(\lambda + \frac{w}{A + \alpha\lambda}\right)\right)\lambda \tag{3.6.7}$$

and the corresponding optimal profit is defined as $\pi(\lambda^\star, \lambda^\star)$. Denote the steady state profit in the myopic policy and the strategic policy as $\pi(\lambda^M, \lambda^M)$ and $\pi(\lambda^{\star\star}, \lambda^{\star\star})$ respectively. We have the following result:

Proposition 3.4. *The steady state arrival rate in the strategic policy is bounded in* $[\lambda^M, \lambda^\star]$*, i.e.,* $\lambda^M \leq \lambda^{\star\star} \leq \lambda^\star$*. Moreover, the profit function is bounded in* $[\pi(\lambda^M, \lambda^M), \pi(\lambda^{\star\star}, \lambda^{\star\star})]$*, i.e.,* $\pi(\lambda^M, \lambda^M) \leq \pi(\lambda^{\star\star}, \lambda^{\star\star}) \leq \pi(\lambda^\star, \lambda^\star)$*.*

Therefore, from the above result, we can see under the fixed price constraint, compared with the optimal arrival rate and profit in the static single period model, the steady state results in the strategic policy are smaller. In the dynamic model, consumer purchase decisions depend on the arrival rate as the existing customer base in the previous period, and there exists delay in the social interaction effect. However, in the static single period model, this delay effect does not exist. Therefore, the feasible arrival rate space for the monopolist to make decisions is larger, indicating an even higher profit. The above result also provides another way for the monopolist

Tab. 3.2: Comparison between strategic policy and myopic policy.

Steady state arrival rate	Steady state service rate	Steady state profit
$\lambda^{\star\star} > \lambda^M$	$\mu^{\star\star} >< \mu^M$	$\pi^{\star\star} > \pi^M$

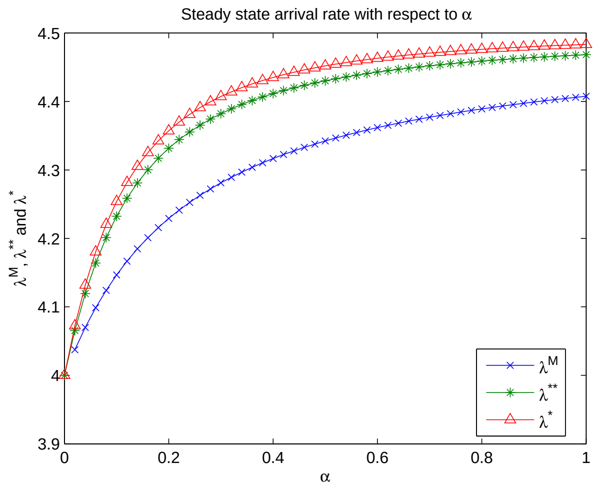

Fig. 3.6.5: Steady state arrival rate with respect to the social interaction intensity $\alpha \in [0, 1]$ under three different policies.

to estimate the steady state arrival rate in the strategic policy, i.e., how much of the market should be covered in the long-run. The monopolist can set the steady state service rate based on the myopic policy and the static single period model, which may be an appropriate service rate decision, especially when the discount factor is not known, for example $\mu^s = \lambda^s + \frac{w}{A+\alpha\lambda^s}$, where $\lambda^s = \theta\lambda^\star + (1-\theta)\lambda^M$ and $\theta \in [0, 1]$ can measure the magnitude of the discount factor. If the monopolist cares more about the current profit, a small θ can be chosen; otherwise, a large θ should be preferred.

Table 3.2 summarizes the comparison between strategic policy and myopic policy in terms of the steady state arrival rate, service rate and profit.

Fig.3.6.5 shows the steady state arrival rate with respect to α, with the parameters $v = 10,\ p = 9,\ \beta = 1,\ w = 1,\ \delta = 0.8$; Fig.3.6.6 shows the steady state arrival rate with respect to δ, with the parameters $v = 10,\ p = 9,\ \beta = 1,\ w = 1,\ \alpha = 0.8$, where the other two steady state arrival rates are independent of δ.

3.7 Dynamic Pricing Strategy Under Social Interactions

3.7.1 Model Settings

In this section, we consider the dynamic pricing strategy under the constant service rate constraint. Service rate can be considered as one aspect of service quality in

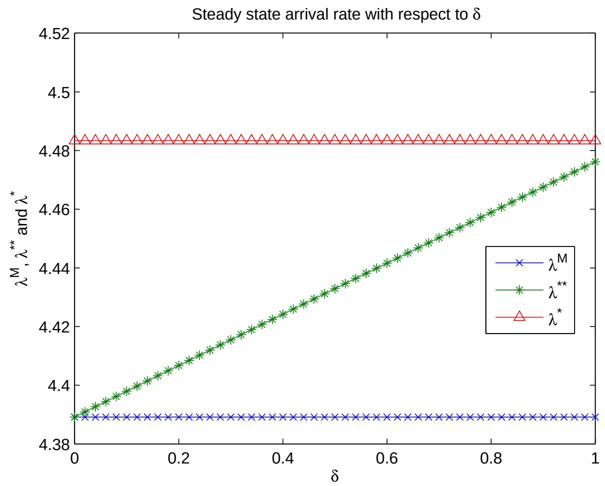

Fig. 3.6.6: Steady state arrival rate with respect to the discount factor $\delta \in [0,1]$ under three different policies.

some service industries, for example, the consumer-intensive service. Therefore, the constant service rate can be considered as the fixed or standard service quality due to regulations or other factors, such as the quality promise of the service provider. While we assume the price can be changed in each period, the monopolist decides the optimal price in order to maximize the long-run discounted profit. We make the following assumption in this section:

Assumption under constant service rate policy: $\mu B > w$*, where* $B = v - \beta\mu > 0$ *is the net value of the service from the perspective of the service provider.*

The assumption indicates, under the fixed service rate μ, if the price p is charged as low as the service rate cost, consumers have incentives to join the queue to purchase the service even if there is no social interaction effect (there are no consumers purchasing the service in previous periods), since $\mu B > w \Rightarrow v - \beta\mu - \frac{w}{\mu} > 0$.

In each period, the equilibrium arrival rate will drive consumer surplus to be zero. Therefore, we have the state transition in terms of the equilibrium arrival rate in period t as

$$v + \alpha\lambda_{t-1} - p_t - CW(\lambda_t, \mu) = 0 \Rightarrow \lambda_t = \mu - \frac{w}{v + \alpha\lambda_{t-1} - p_t}$$

where the price p_t is bounded in set $p_t \in P_t = \left[\beta\mu, v + \alpha\lambda_{t-1} - \frac{w}{\mu}\right]$, and the arrival rate is also bounded as $0 \leq \lambda_t < \mu$.

Given the price decision, the profit in each period is

$$\pi(\lambda_{t-1}, p_t) = (p_t - \beta\mu)\lambda_t = (p_t - \beta\mu)\left(\mu - \frac{w}{v + \alpha\lambda_{t-1} - p_t}\right) \tag{3.7.1}$$

which is increasing in λ_{t-1}, since the derivative with respect to λ_{t-1} is $\frac{(p_t-\beta\mu)\alpha w}{(v+\alpha\lambda_{t-1}-p_t)^2} > 0$. For the fixed service rate μ, the profit is definitely bounded, since it is concave in p_t in a bounded set. For an initial arrival rate λ_0, the monopolist determines the price in each period to maximize the long-run discounted profit as

$$V(\lambda_0) = \sup_{p_t \in P_t} \sum_{t=1}^{\infty} \delta^{t-1} \pi(\lambda_{t-1}, p_t), \ \ s.t.\ \lambda_t = \mu - \frac{w}{v + \alpha\lambda_{t-1} - p_t}, \ \ t \geq 1 \tag{3.7.2}$$

The profit function in each period is concave and bounded, so the value function is the unique bounded solution to the following Bellman equation:

$$V(\lambda) = \sup_{p \in P} W(p, \lambda), \ \text{ where } W(p, \lambda) = \pi(\lambda, p) + \delta V\left(\mu - \frac{w}{v + \alpha\lambda - p}\right) \tag{3.7.3}$$

Since the single period profit function $\pi(\lambda, p)$ is increasing in λ, the value function $V(\lambda)$ is increasing in λ, indicating a large initial market share is always beneficial to the monopolist.

Similar to the dynamic service rate model with constant price, the state transition function is nonlinear in the decision variable p, which may not be easy to derive the property of the policy function. Therefore, we also reformulate the above problem by changing the decision variable as the equilibrium arrival rate, based on the assumption that in each period, the potential market is large enough for the monopolist to cover. Therefore, the monopolist can determine the equilibrium arrival rate in each period by setting the corresponding price. The price is determined by the selected arrival rate in period t as:

$$p_t = v + \alpha\lambda_{t-1} - \frac{w}{\mu - \lambda_t}$$

where the action space of λ_t, denoted as $\hat{\Lambda}_t$ which is bounded and compact, can be solved from the following inequality

$$p_t = v + \alpha\lambda_{t-1} - \frac{w}{\mu - \lambda_t} \geq \beta\mu \Rightarrow \lambda_t \in \Lambda_t = \left[0, \mu - \frac{w}{B + \alpha\lambda_{t-1}}\right]$$

and the single period profit function is reformulated as

$$\pi(\lambda_{t-1}, p_t) = \left(B + \alpha\lambda_{t-1} - \frac{w}{\mu - \lambda_t}\right)\lambda_t = \pi(\lambda_{t-1}, \lambda_t) \tag{3.7.4}$$

where $\pi(\lambda_{t-1}, \lambda_t)$ is concave in λ_t and bounded.

Starting at the initial arrival rate λ_0, the optimal value function is reformulated as

$$V(\lambda_0) = \sup_{\lambda_t \in \Lambda_t} \sum_{t=1}^{\infty} \delta^{t-1} \pi(\lambda_{t-1}, \lambda_t) \tag{3.7.5}$$

which is the unique bounded solution of the following Bellman equation:

$$V(\lambda) = \sup_{\hat{\lambda} \in \hat{\Lambda}} W(\lambda, \hat{\lambda}), \text{ where } W(\lambda, \hat{\lambda}) = \pi(\lambda, \hat{\lambda}) + \delta V(\hat{\lambda}) \tag{3.7.6}$$

The single period profit function $\pi(\lambda, \hat{\lambda})$ is increasing in λ, indicating the value function $V(\lambda)$ is increasing in λ. Therefore, we see the reformulated dynamic programming problem in the pricing strategy is similar to the dynamic service rate problem with the constant price constraint. The difference lies that the state transition mechanisms are different in the two models.

Because the action set $\hat{\Lambda}$ is compact and all functions are continuous, there exists an optimal stationary arrival rate policy that solves the above problem. We define the optimal arrival rate policy as $\hat{\lambda}^\star(\lambda) = \arg\max_{\hat{\lambda}} W(\lambda, \hat{\lambda})$. The arrival rate path $\{\hat{\lambda}_t^\star, t \geq 1\}$ starting with an initial arrival rate λ_0 is computed sequentially as $\hat{\lambda}_t^\star = \hat{\lambda}^\star(\hat{\lambda}_{t-1}^\star)$. The sequence $\{\hat{\lambda}_t^\star\}$ of the optimal decision is also the optimal state path $\{\lambda_t^\star, t \geq 1\}$ in the sequence problem. The corresponding price path $\{p_t^\star, t \geq 1\}$ can be computed through $p_t = v + \alpha\lambda_{t-1} - \frac{w}{\mu - \lambda_t}$ in each period.

3.7.2 Myopic Policy

Similar to the previous section, we first investigate the dynamic pricing decision in the myopic policy, which is denoted by the superscript M. Therefore, the arrival rate path and price path are denoted as $\{\lambda_t^M, t \geq 1\}$ and $\{p_t^M, t \geq 1\}$ respectively. Similar to the previous section, we have the supermodularity of the profit function in each period and the increasing property of the optimal myopic arrival rate decision:

Lemma 3.5. *The single period profit is supermodular in* $(\lambda, \hat{\lambda})$*, and the profit maximizing arrival rate* $\tilde{\lambda}(\lambda) = \arg\max_{\hat{\lambda} \in \hat{\Lambda}} \pi(\lambda, \hat{\lambda})$ *is increasing in* λ*. Correspondingly, the single period profit maximizing price* $\tilde{p}(\lambda) = v + \alpha\lambda - \frac{w}{\mu - \tilde{\lambda}(\lambda)}$ *is also increasing in* λ*.*

Therefore, a larger existing customer base will always lead to a higher price in the myopic policy, and the corresponding effective arrival rate is also increasing in λ. We have the following sequence of arrival rate and corresponding price decision in the myopic policy as

$$\lambda_t^M = \mu - \sqrt{\frac{w\mu}{B + \alpha\lambda_{t-1}^M}}, \; p_t^M = v + \alpha\lambda_{t-1}^M - \sqrt{\frac{w(B + \alpha\lambda_{t-1}^M)}{\mu}}$$

Since $\tilde{\lambda}(\lambda)$ is increasing and the arrival rate space is bounded, λ_t^M is increasing in λ_{t-1}^M and there exists a unique steady state λ^M of the sequence $\{\lambda_t^M, t \geq 1\}$ which is the solution to the cubic function $\lambda = \mu - \sqrt{\frac{w\mu}{B+\alpha\lambda}}$. Correspondingly, the steady state price in the myopic policy is $p^M = v + \alpha\lambda^M - \sqrt{\frac{w(B+\alpha\lambda^M)}{\mu}}$.

Lemma 3.6. *There exists a unique steady state arrival rate λ^M in the myopic policy in $(0, \mu)$, i.e., $\lambda^M \in (0, \mu)$, thus a unique steady state price p^M.*

Proof. Define $F(\lambda) = (\mu - \lambda)^2(B + \alpha\lambda) - w\mu$. There are three solutions, namely $\lambda^{(i)}, i = 1, 2, 3$ to the equation $F(\lambda) = 0$. We have $F(0) = \mu^2 B - w\mu > 0$ based on the assumption $\mu B > w$, and $F(\mu) = -w\mu < 0$. We also have $\forall\lambda \gg \mu$, $F(\lambda) > 0$ and $\forall\lambda \leq -\frac{B}{\alpha}$, $F(\lambda) < 0$. Therefore the three solutions satisfy $-\frac{B}{\alpha} < \lambda^{(1)} < 0 < \lambda^{(2)} < \mu < \lambda^{(3)}$, indicating the unique steady state arrival rate in the myopic policy is $\lambda^M = \lambda^{(2)} \in (0, \mu)$. □

Starting with the initial market size λ^M, the myopic policy will always set the price at p^M, which leads to the same arrival rate λ^M in each period. We have the following result in terms of the monotonicity of the arrival rate path and the price path in the myopic policy:

Lemma 3.7. *For any initial market size λ_0, the arrival rate path $\{\lambda_t^M\}$ and the price path $\{p_t^M\}$ monotonically converge to λ^M and p^M respectively in the same direction in the myopic policy.*

Therefore, starting with a larger initial market size $\lambda_0 > \lambda^M$, the monopolist will charge a higher price and the arrival rate will be reduced gradually which will converge to the unique steady state; due to the decreasing arrival rate path, the price path is also decreasing. If the initial market size is small as $\lambda_0 < \lambda^M$, the monopolist will charge a lower price and the arrival rate paths will monotonically converge to the steady state result increasingly; due to the increasing arrival rate path, the price path is also increasing. The myopic policy is equivalent to setting the discount factor $\delta = 0$, i.e., the manager ignores the impact of the current decision on the arrival rate in the future due to the influence of social interactions. Therefore, compared with the strategic policy, the myopic policy will select a smaller arrival rate by charging a higher price in each period, as shown in the following result:

Proposition 3.5. *The arrival rate decision in the myopic policy is smaller than that in the strategic policy, i.e., $\tilde{\lambda}(\lambda) \leq \hat{\lambda}^\star(\lambda)$, while the price satisfies $\tilde{p}(\lambda) \geq \hat{p}^\star(\lambda)$, where $\tilde{p}(\lambda)$ and $\hat{p}^\star(\lambda)$ are the corresponding price in the myopic policy and the strategic* *policy. Moreover, for any initial arrival rate λ_0, the arrival rate in the myopic policy at any point in time is less than that in the strategic policy, i.e.,* $\lambda_t^M \leq \lambda_t^\star$.

Proof. Since $V(\lambda)$ is increasing, we have $\hat{\lambda}^\star(\lambda) = \arg\max \left\{ \pi(\lambda, \hat{\lambda}) + \delta V(\hat{\lambda}) \right\} \geq \arg\max \pi(\lambda, \hat{\lambda}) = \tilde{\lambda}(\lambda)$. We also have $\tilde{p}(\lambda) = v + \alpha\lambda - \frac{w}{\mu - \tilde{\lambda}(\lambda)} \geq v + \alpha\lambda - \frac{w}{\mu - \hat{\lambda}^\star(\lambda)} = \hat{p}^\star(\lambda)$. For any initial arrival rate λ_0, we have $\lambda_1^\star = \hat{\lambda}^\star(\lambda_0) \geq \tilde{\lambda}(\lambda_0) = \lambda_1^M$ and $p_1^M = \tilde{p}(\lambda_0) \geq \hat{p}^\star(\lambda_0) = p_1^\star$. We use induction to prove the rest. Suppose $\lambda_t^\star \geq \lambda_t^M$. We have $\lambda_{t+1}^\star = \hat{\lambda}^\star(\lambda_t^\star) \geq \tilde{\lambda}(\lambda_t^\star) \geq \tilde{\lambda}(\lambda_t^M) = \lambda_{t+1}^M$ based on the increasing property of $\tilde{\lambda}(\lambda)$. □

The above result indicates, given the same existing customer base λ, compared with the strategic policy, the myopic policy will always charge a higher price, which leads to a lower arrival rate. Thus, ignoring the influence of social interactions in the future periods, the myopic policy always overprices the service, while the strategic policy trades off the current period profit by charging a lower price followed by a larger arrival rate in the current period for a higher long term benefit. However, we can not determine the relation on the price paths in the two policies, $p_t^\star$ and p_t^M, starting at the same initial market size λ_0. It is possible that with the time period increasing, the price charged by the strategic policy can be higher than that in the myopic policy, when the existing customer base has been built up due to the influence of social interactions. Given the fixed capacity, the lower price in the strategic policy explains why some companies intentionally under-price their products or service, i.e., a lower price can help attract more demand through social interactions, which will bring more benefit in the future.

3.7.3 Strategic Policy

Since there exists a unique steady state in the myopic policy both for arrival rate and price, we investigate whether the steady state in the strategic policy exists or not. Similar to the dynamic capacity management model, we first have the monotonicity of the optimal arrival rate in the strategic policy:

Lemma 3.8. *The optimal arrival rate $\hat{\lambda}^\star(\lambda)$ in the strategic policy is increasing in λ. Moreover, the optimal arrival rate path $\lambda_t^\star$ is monotonic.*

Although the optimal arrival rate path is monotonic, the price path is not necessarily monotonic. It is possible that the optimal price path may fluctuate depending on other parameters. Based on the monotonicity of the arrival rate path, in terms of the existence and uniqueness of the steady state arrival rate, similar to the result in the dynamic capacity management model, we have the following result:

Proposition 3.6. *There exists a unique steady state arrival rate $\lambda^{\star\star}$ in the strategic policy. All arrival rate paths converge to the steady state $\lambda^{\star\star}$ monotonically.*

The steady state arrival rate and the price decision in the strategic policy can be solved from the following result:

Theorem 3.2. *In the strategic policy, there exists a unique steady state $\lambda^{\star\star}$, which is the unique solution in $(0, \mu)$ to the cubic function $(B + (1+\delta)\alpha\lambda)(\mu-\lambda)^2 - w\mu = 0$. The steady state price is $p^{\star\star} = v + \alpha\lambda^{\star\star} - \frac{w}{\mu-\lambda^{\star\star}}$. The steady state $\lambda^{\star\star}$ in the strategic policy is increasing in α and δ respectively. Moreover, $\lambda^{\star\star}$ is larger than the steady state in the myopic policy λ^M.*

The result is similar to that in the dynamic capacity management model. The steady state arrival rate in the strategic policy in the dynamic pricing model can be easily solved from a cubic function. The monotonicity of the steady state arrival rate in terms of the social interaction intensity, and the discount factor is similar to the dynamic service rate model with constant price. The strategic policy can always lead to a larger steady state arrival rate than the myopic policy, i.e., a larger market will be covered under the strategic policy. A larger social interaction intensity or a larger discount factor will lead to a larger steady state arrival rate in both policies. Similarly, the queue length, expected waiting time and the utilization under the strategic policy are larger than those under the myopic policy respectively.

Fig.3.7.1-3.7.4 depict the value function, the arrival rate path and the price path, as well as the arrival rate decision in the strategic policy and the myopic policy, with the parameters $v = 10,\ \mu = 7,\ \beta = 1,\ w = 1,\ \alpha = 0.8,\ \delta = 0.9$.

From the comparison on the arrival rate path and the arrival rate decision, we can see the strategic policy always selects a higher arrival rate in order to utilize the influence of social interactions. The price comparison indicates, the strategic policy may first charge a lower price to increase the customer base in the initial periods. However, when the customer base has been built up, the strategic policy will charge a higher price than the myopic policy. Many firms first adopt penetration pricing strategy with a lower price through promotions or other strategies to promote the sales of their products or services in the introduction stage, and then charge the normal or even a higher price after the market base has been built up. Our result can explain this practice, since the customer base has been established, and through WOM, OL, the potential demand will be sustained to be large enough even if companies start to charge a higher price.

Similar to the previous section, the following result provides a sufficient condition for the comparison on the steady sate price level in the myopic policy and the strategic policy:

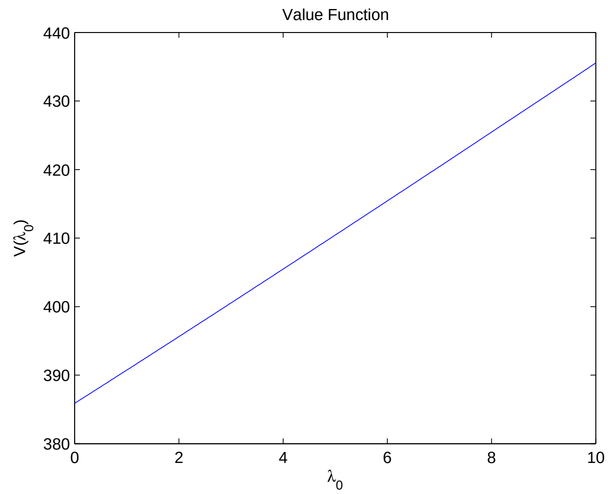

Fig. 3.7.1: Value function in the dynamic pricing strategy with respect to the initial arrival rate.

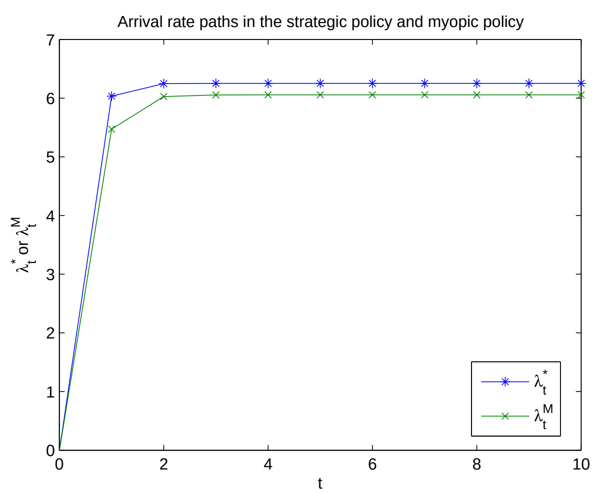

Fig. 3.7.2: Arrival rate paths in the dynamic pricing strategy under the myopic policy and the strategic policy.

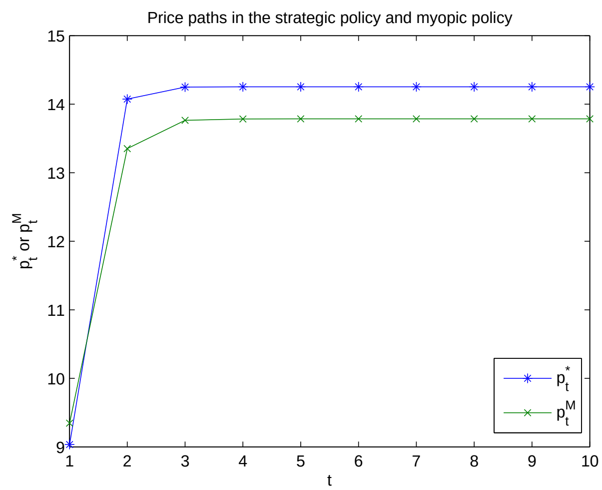

Fig. 3.7.3: Price paths in the dynamic pricing strategy under the myopic policy and the strategic policy.

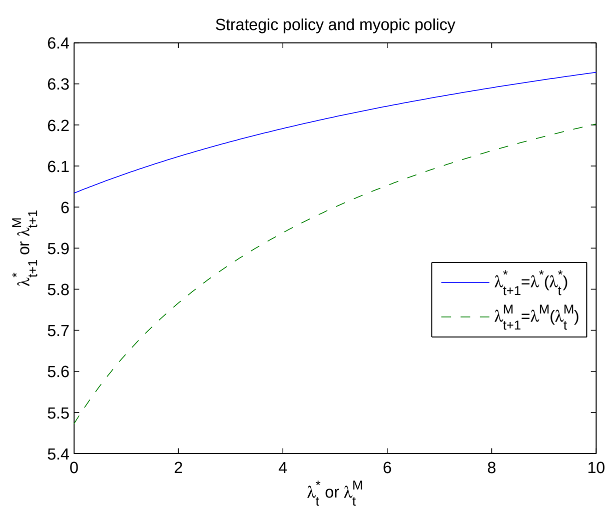

Fig. 3.7.4: Arrival rate dynamics in the dynamic pricing strategy under the myopic policy and the strategic policy.

Corollary 3.2. *If* $\alpha \le \frac{w}{(\mu-\lambda^M)^2}$*,* $p^{\star\star} \le p^M$*; while if* $\alpha > \frac{w}{(\mu-\lambda^{\star\star})^2}$*,* $p^{\star\star} > p^M$*.*

Therefore, the social interaction intensity impacts the steady state price level. Specifically, if the social interaction intensity is small, the steady state price in the strategic policy will be smaller than that in the myopic policy. Under a small social interaction intensity, the steady state price is always decreasing in the arrival rate, and the strategic policy sacrifices the profit margin in order to maintain a larger arrival rate. However, if the social interaction intensity is large, the steady state price in the strategic policy can be larger than that in the myopic policy. The reason is that, in the strategic policy, the steady state arrival rate is larger than that in the myopic policy, which leads to a higher price level. In another words, the monopolist can eventually get a higher profit margin from the large customer base lead by low prices during previous periods.

Although the comparison on the steady state prices may depend on the social interaction intensity and other parameters, the profit in the steady state is comparable. We consider the optimal arrival rate under fixed service rate in the static single period model, where the price is $v + \alpha\lambda - p - \frac{w}{\mu-\lambda} = 0 \Rightarrow p = v + \alpha\lambda - \frac{w}{\mu-\lambda}$ with the profit function as

$$\pi(\lambda, \lambda) = \left(v + \alpha\lambda - \frac{w}{\mu-\lambda} - \beta\mu\right)\lambda = \left(B + \alpha\lambda - \frac{w}{\mu-\lambda}\right)\lambda \tag{3.7.7}$$

which admits an optimal arrival rate denoted as $\lambda^\star$, which is the unique solution to the following cubic function

$$B + 2\alpha\lambda - \frac{w\mu}{(\mu-\lambda)^2} = 0 \Rightarrow (B + 2\alpha\lambda)(\mu-\lambda)^2 - w\mu = 0$$

Denote the optimal profit in the static single period model as $\pi(\lambda^\star, \lambda^\star)$ and the steady state profit levels in the myopic policy and the strategic policy as $\pi(\lambda^M, \lambda^M)$ and $\pi(\lambda^{\star\star}, \lambda^{\star\star})$ respectively. We have the following result:

Proposition 3.7. *The steady state arrival rate in the strategic policy is bounded in* $[\lambda^M, \lambda^\star]$*, i.e.,* $\lambda^M \le \lambda^{\star\star} \le \lambda^\star$*. Moreover, the profit function is bounded in* $\left[\pi(\lambda^M, \lambda^M), \pi(\lambda^\star, \lambda^\star)\right]$*, i.e.,* $\pi(\lambda^M, \lambda^M) \le \pi(\lambda^{\star\star}, \lambda^{\star\star}) \le \pi(\lambda^\star, \lambda^\star)$*.*

Therefore, from the above result, we can see under the same service rate, compared with the optimal arrival rate and profit in the static single period model, the steady state in the strategic policy is less. The reason is due to the time delay in the social interaction effect, which reduces the action space in the dynamic model. The above result also provides an alternative way for the monopolist to estimate the steady state arrival rate and the steady state price level in the strategic policy.

Tab. 3.3: Comparison between strategic policy and myopic policy.

Steady state arrival rate	Steady state price	Steady state profit
$\lambda^{\star\star} > \lambda^M$	$p^{\star\star} >< p^M$	$\pi^{\star\star} > \pi^M$

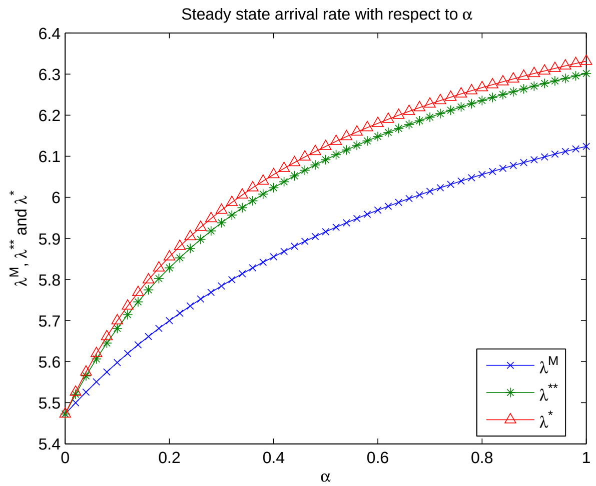

Fig. 3.7.5: Steady state arrival rate with respect to the social interaction intensity $\alpha \in [0, 1]$ under three different policies.

The monopolist can set the steady state price based on the myopic policy and the static single period model, which may be an appropriate price decision, especially when the discount factor is not certain, for example $p^s = v + \alpha\lambda^s - \frac{w}{\mu - \lambda^s}$, where $\lambda^s = \theta\lambda^\star + (1-\theta)\lambda^M$ and $\theta \in [0, 1]$ can measure the magnitude of the discount factor. If the monopolist cares more about the current profit, a small θ can be chosen; otherwise, a large θ should be preferred.

Table 3.3 summarizes the comparison between strategic policy and myopic policy in terms of the steady state arrival rate, price and profit.

Fig.3.7.5 shows the steady state arrival rate with respect to α, with the parameters $v = 10,\ \mu = 7,\ \beta = 1,\ w = 1,\ \delta = 0.8$; Fig.3.7.6 shows the steady state arrival rate with respect to δ, with the parameters $v = 10,\ \mu = 7,\ \beta = 1,\ w = 1,\ \alpha = 0.8$, where the other two steady state arrival rates are independent of δ.

3.8 Dynamic Capacity and Pricing Strategy

In this section, we consider the dynamic service rate and pricing strategy in a general model, where the service rate and the price are simultaneously determined in each period. Suppose the total potential arrival rate is Λ in each period. Based on the arrival rate λ_{t-1}, given the service rate μ_t and price p_t, the equilibrium arrival rate

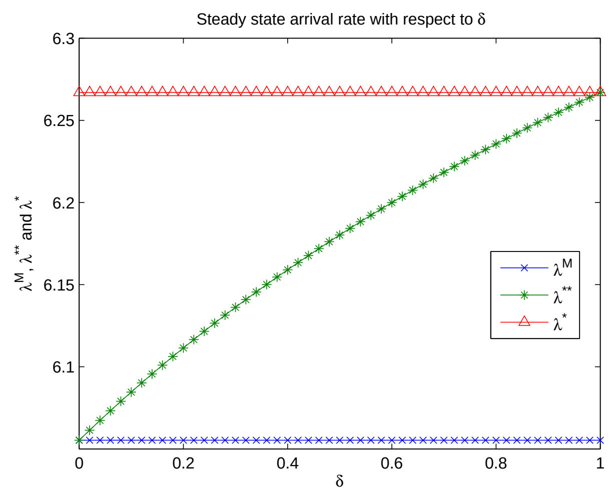

Fig. 3.7.6: Steady state arrival rate with respect to the discount factor $\delta \in [0, 1]$ under three different policies.

satisfies the state transition function

$$\lambda_t = \min\left(\mu_t - \frac{w}{v + \alpha\lambda_{t-1} - p_t}, \Lambda\right)$$

and the single period profit is

$$\pi(\mu_t, p_t, \lambda_{t-1}) = (p_t - \beta\mu_t)\min\left(\mu_t - \frac{w}{v + \alpha\lambda_{t-1} - p_t}, \Lambda\right) \tag{3.8.1}$$

The long-run discounted profit with the initial arrival rate λ_0 and discount factor $\delta \in (0, 1)$ under the policy $(\mu_{\mathbf{t}}, \mathbf{p_t})$ is formulated as

$$\begin{aligned} \Pi(\lambda_0) \quad &= \quad \sum_{t=0}^{\infty} \delta^t \pi(\mu_{t+1}, p_{t+1}, \lambda_t) \qquad (3.8.2) \\ s.t. \qquad & \lambda_t = \min\left(\mu_t - \frac{w}{v + \alpha\lambda_{t-1} - p_t}, \Lambda\right), \ t \geq 1 \end{aligned}$$

The objective of the monopolist is to decide the optimal service rate and price policy to maximize the long-run discounted profit as

$$V(\lambda_0) = \sup_{\mu_t, p_t} \Pi(\lambda_0)$$

where the value function $V(\lambda)$ is the unique bounded solution to the following Bellman equation:

$$V(\lambda) = \sup_{\mu,p} \pi(\mu,p,\lambda) + \delta V\left(\min\left(\mu - \frac{w}{v+\alpha\lambda - p}, \Lambda\right)\right) \tag{3.8.3}$$

In the following section, we consider the myopic policy and the strategic policy respectively and compare the dynamic behavior of the system under each policy. The decisions and state variables in the myopic policy are denoted as $\{\mu_t^M, p_t^M, \lambda_t^M\}$, while in the strategic policy as $\{\mu_t^\star, p_t^\star, \lambda_t^\star\}$.

It can be shown that if the social interaction intensity is larger than or approaching the unit service rate cost $\alpha \geq \beta$ or $\alpha \to \beta$, the monopolist will cover the whole market eventually even under the myopic policy. In the following section, to investigate the influence of social interactions on the service rate and price decisions, we focus on the case where the social interaction intensity is not large, i.e., $\alpha < \beta$. In order to simplify the analysis, we also assume the total potential arrival rate Λ is large enough in each period, such that the equilibrium arrival rate in each period is smaller than Λ under any feasible policy.

3.8.1 Myopic Policy

Given the arrival rate λ_{t-1}, we first solve the single period profit optimization problem as

$$\max_{\mu_t,p_t} \pi(\mu_t,p_t,\lambda_{t-1}) = (p_t - \beta\mu_t)\left(\mu_t - \frac{w}{v+\alpha\lambda_{t-1}-p_t}\right) \tag{3.8.4}$$

where the profit function $\pi(\mu_t,p_t,\lambda_{t-1})$ is concave in μ_t and p_t respectively and supermodular in (μ_t,p_t). To solve the optimal service rate and price, we first solve the optimal p (or μ) for fixed μ (or p), and than solve the optimal μ (or p). Thus, we define

$$\max_{\mu_t,p_t} \pi(\mu_t,p_t,\lambda_{t-1}) \equiv \max_{p_t}\left\{\max_{\mu_t} \pi(\mu_t,p_t,\lambda_{t-1})\right\} \tag{3.8.5}$$

We first solve the optimal service rate given the price p_t as $\mu(p_t)$, and then solve the optimal price as p_t^M. The optimal service rate and pricing decisions will be $\left(\mu(p_t^M), p_t^M\right)$, or $\left(\mu_t^M(\lambda_{t-1}), p_t^M(\lambda_{t-1})\right)$ in terms of the existing arrival rate. Specifically, we have

$$p_t^M(\lambda_{t-1}) = v + \alpha\lambda_{t-1} - \sqrt{\beta w},\ \mu(p_t^M) = \frac{p_t}{2\beta} + \frac{w}{2(v+\alpha\lambda_{t-1}-p_t)}|_{p_t^M} = \mu_t^M(\lambda_{t-1})$$

and the corresponding equilibrium arrival rate in period t denoted as $\lambda_t^M(\lambda_{t-1})$ is

$$\lambda_t^M(\lambda_{t-1}) = \mu_t - \frac{w}{v+\alpha\lambda_{t-1}-p_t} = \frac{v+\alpha\lambda_{t-1}}{2\beta} - \sqrt{\frac{w}{\beta}} = \frac{v+\alpha\lambda_{t-1} - 2\sqrt{\beta w}}{2\beta}$$

which is increasing in λ_{t-1}. Therefore, if the total potential arrival rate is small in each period, such that $\frac{v}{2\beta} - \sqrt{\frac{w}{\beta}} \geq \Lambda$, the optimal strategy is to cover the whole market in each period. Besides, if $\frac{\alpha}{2\beta} \geq 1$, i.e., the social interaction intensity is large enough, the arrival rate will increase to Λ after several transient periods. Therefore, after the transient stage, the optimal strategy is to cover the whole market and grab all consumer surplus. In the following section, we assume $v \geq 2\sqrt{\beta w}$ to guarantee that the optimal arrival rate is always non-negative. The optimal service rate and price under the full market coverage is given as the following result:

$$\max_{\mu_t, p_t}(p_t - \beta\mu_t)\Lambda_t, \text{ s.t. } v + \alpha\lambda_{t-1} - \frac{w}{\mu_t - \Lambda} - p_t = 0 \tag{3.8.6}$$

where we can solve the optimal service rate and price as

$$\mu_t^M = \Lambda_t + \sqrt{\frac{w}{\beta}}, \; p_t^M = v + \alpha\lambda_{t-1} - \beta\Lambda - 2\sqrt{\beta w}$$

Therefore, we have the following result in terms of the myopic policy with a larger social interaction effect:

Proposition 3.8. *Suppose the total potential market size in each period is Λ. If the social interaction intensity is large, such that $\alpha \geq 2\beta$, the market will be fully covered after transient stages (if $\frac{v}{2\beta} - \sqrt{\frac{w}{\beta}} \geq \Lambda$, there is no transient stage at all) with the optimal service rate and price as $\mu_t^M = \Lambda + \sqrt{\frac{w}{\beta}}$, $p_t^M = v + (\alpha - \beta)\Lambda - 2\sqrt{\beta w}$.*

Remark 3.2. After the transient stage, the myopic policy is identical to the strategic policy since the monopolist covers the whole market.

Therefore, in the following section, for the myopic policy, to reduce the complexity of the analysis and exclude the full market coverage situation, we focus on the case with $\alpha < 2\beta$ and the total potential arrival rate Λ is large enough for the monopolist to cover in each period. We focus on the dynamics of the arrival rate and the property of the optimal service rate and price decisions in the myopic policy. The steady state arrival rate (if exists) denoted as λ^M is given as

$$\lambda^M = \frac{v - 2\sqrt{\beta w}}{2\beta - \alpha}$$

which is unique. The corresponding service rate and price in the steady state are

$$\mu^M = \frac{v + \alpha\lambda^M}{2\beta} = \frac{v}{2\beta} + \frac{\alpha}{2\beta}\left(\frac{v - 2\sqrt{\beta w}}{2\beta - \alpha}\right), \; p^M = v + \alpha\left(\frac{v - 2\sqrt{\beta w}}{2\beta - \alpha}\right) - \sqrt{\beta w}$$

In terms of the existence of the steady state in the myopic policy, we have the following result:

Lemma 3.9. *(Steady state in the myopic policy) Given any initial arrival rate λ_0, there exists a unique steady state λ^M, where the arrival rate path converges to λ^M monotonically; the corresponding service rate path and the price path also monotonically converge to μ^M and p^M respectively.*

Proof. If $\lambda_0 > \lambda^M$, the arrival rate dynamics in the myopic policy indicates $\lambda^M < \lambda_1^M < \lambda_0$. The corresponding service rate and price in period 1 and period 2 satisfy $\mu_1^M(\lambda_0) \geq \mu_2^M(\lambda_1^M)$ and $p_1^M(\lambda_0) \geq p_2^M(\lambda_1^M)$ respectively followed from the service rate and the price dynamics. Then using induction, we can prove that the service rate path, the price path and the arrival rate path converge to the corresponding steady state respectively in a decreasing order. If $\lambda_0 < \lambda^M$, the service rate path, the price rate path and the arrival rate path will converge to the corresponding steady state respectively in an increasing order. $\square$

From the steady state service rate and price in the myopic policy, we can see the steady state decisions are both increasing in the social interaction intensity α. The profit margin from each served customer in the steady state is $p^M - \beta\mu^M = \frac{v}{2} + \frac{\alpha}{2\beta}\left(\frac{v-2\sqrt{\beta w}}{2\beta-\alpha}\right) - \sqrt{\beta w}$, which is also increasing in α. Therefore, in the myopic policy, although the service rate and price may simultaneously increase gradually if the initial market size is small, the profit margin will always increase in the social interaction intensity. The increased profit margin and the market size lead to an increased profit. In practice, some firms provide a higher speed or higher quality service and also charge a higher price with a higher profit margin, compared with their competitors. The influence of social interactions may provide an explanation for this market segmentation, i.e., those firms with a higher price and a higher service speed cover a larger market due to the higher influence of social interactions.

3.8.2 Strategic Policy

We compare the operational decisions and the steady state in the myopic policy and the strategic policy. We first investigate whether there exists a steady state in the strategic policy. Since in each period, there are two decision variables and the state transition function in terms of the equilibrium arrival rate is nonlinear, directly investigating the optimal service rate and price policy as well as the steady state may not be straightforward. In the following section, we first consider a finite-horizon problem with $N \geq 2$ periods and study its structural property.

We start with a two-period problem with the initial arrival rate λ_0. The monopolist decides the service rate and price in period 1 and 2. We use the backward induction procedure to solve the optimal service rate and price decisions in the two-period problem. Compared with the myopic policy, we have the following relationship in terms of the price, service rate and the arrival rate in period 1 and period 2 as

$$\begin{aligned} p_1^\star(\lambda_0) &= p_1^M(\lambda_0),\ \mu_1^\star(\lambda_0) = \frac{v+\alpha\lambda_0}{2\beta} + \frac{\delta\alpha}{4\beta} = \mu_1^M(\lambda_0) + \frac{\delta\alpha}{4\beta}, \\ \lambda_1^\star(\lambda_0) &= \frac{v+\alpha\lambda_0 - 2\sqrt{\beta w}}{2\beta} + \frac{\delta\alpha}{4\beta} = \lambda_1^M(\lambda_0) + \frac{\delta\alpha}{4\beta} \end{aligned}$$

$$p_2^\star(\lambda_1^\star) > p_2^M(\lambda_1^M),\ \mu_2^\star(\lambda_1^\star) = \frac{v+\alpha\lambda_1}{2\beta} = \mu_2^M(\lambda_1^M) + \frac{\alpha}{2\beta}\frac{\delta\alpha}{4\beta},\ \lambda_2^\star(\lambda_1^\star) = \lambda_2^M(\lambda_1^M) + \frac{\alpha}{2\beta}\frac{\delta\alpha}{4\beta}$$

Therefore, the monopolist should increase the period 1 service rate in the strategic policy to induce a higher equilibrium arrival rate which serves as the consumer base in period 2. The service rate and arrival rate in period 2 under strategic policy are larger than those in the myopic policy. Although the price in period 1 is the same in the myopic policy, the price in period 2 is larger than that in the myopic policy.

For the finite-horizon problem with N periods, the backward induction procedure indicates in the last period, the monopolist always adopts the myopic policy. In period 1, we can see the service rate in the strategic policy has an additional constant term $\frac{\delta\alpha}{4\beta}$, which leads to a higher arrival rate with the same additional term in period 1. Then the price and the arrival rate will be larger in the following periods compared with the myopic policy. Formally, we have the following result in terms of the comparison on the service rate, price and the arrival rate in the strategic policy and the myopic policy:

Theorem 3.3. *For the finite-horizon problem of $N \geq 2$ periods with the initial arrival rate λ_0, the comparisons between the service rate path, price path and the arrival rate path in the strategic policy and the myopic policy are given as the following:*

- *the service rate comparison is $\forall t \in \{1, \cdots, N-1\}$,*

$$\mu_t^\star = \mu_t^M + \frac{\delta\alpha}{4\beta}\left(\frac{1-(\frac{\alpha}{2\beta})^t}{1-\frac{\alpha}{2\beta}}\right),\ \mu_N^\star = \mu_N^M + \frac{\alpha}{2\beta}\frac{\delta\alpha}{4\beta}\left(\frac{1-(\frac{\alpha}{2\beta})^{N-1}}{1-\frac{\alpha}{2\beta}}\right);$$

- *the price comparison is*

$$p_1^\star = p_1^M,\ \forall t \in \{2, \cdots, N\},\ p_t^\star = p_t^M + \alpha\left[\frac{\delta\alpha}{4\beta}\left(\frac{1-(\frac{\alpha}{2\beta})^{t-1}}{1-\frac{\alpha}{2\beta}}\right)\right];$$

- *the arrival rate comparison is* $\forall t \in \{1, \cdots, N-1\}$,

$$\lambda_t^\star = \lambda_t^M + \frac{\delta\alpha}{4\beta}\left(\frac{1-(\frac{\alpha}{2\beta})^{t}}{1-\frac{\alpha}{2\beta}}\right),\ \lambda_N^\star = \lambda_N^M + \frac{\alpha}{2\beta}\frac{\delta\alpha}{4\beta}\left(\frac{1-(\frac{\alpha}{2\beta})^{N-1}}{1-\frac{\alpha}{2\beta}}\right).$$

For the finite-horizon problem, we have the following immediate conclusion: *the service rate and the price in the strategic policy are always larger than the corresponding decisions in the myopic policy.* The managerial implication for the above result is that in order to optimize the long-run discounted profit, the monopolist should adopt a larger service rate, and a larger price in order to induce a larger arrival rate base which will benefit the monopolist in future periods due to the influence of social interactions, since the arrival rate in the strategic policy is always larger than that in the myopic policy.

We compare the single period profit in the strategic policy and the myopic policy for the finite-horizon problem with N periods. Clearly, in the last period N, we have $\pi_N^\star(\mu_N^\star, p_N^\star, \lambda_N^\star) > \pi_N^M(\mu_N^M, p_N^M, \lambda_N^M)$, since the optimal profit function is increasing in the arrival rate base and we have $\lambda_{N-1}^\star > \lambda_{N-1}^M$. In the first period, we have $\pi_1^\star(\mu_1^\star, p_1^\star, \lambda_1^\star) < \pi_1^M(\mu_1^M, p_1^M, \lambda_1^M)$, since myopic policy always maximizes the current period profit. For the period $t \in \{2, \cdots, N-1\}$, substituting the above relationship, after simplification, we have

$$\begin{aligned} &\pi_t^\star(\mu_t^\star, p_t^\star, \lambda_t^\star) = (p_t^\star - \beta\mu_t^\star)\lambda_t^\star \\ =& \left(p_t^M - \beta\mu_t^M + (\alpha-\beta)\frac{\delta\alpha}{4\beta} - \frac{\delta\alpha}{4}(\frac{\alpha}{2\beta})^t\right)\left(\lambda_t^M + \frac{\delta\alpha}{4\beta}\left(\frac{1-(\frac{\alpha}{2\beta})^{t}}{1-\frac{\alpha}{2\beta}}\right)\right) \end{aligned} \tag{3.8.7}$$

Therefore, there exists some period t_1, such that if $t < t_1$, we have $\pi_t^\star(\mu_t^\star, p_t^\star, \lambda_t^\star) \leq \pi_t^M(\mu_t^M, p_t^M, \lambda_t^M)$; while if $t \geq t_c$, $\pi_t^\star(\mu_t^\star, p_t^\star, \lambda_t^\star) > \pi_t^M(\mu_t^M, p_t^M, \lambda_t^M)$.

In terms of the comparison on the profit margin, the gap between the strategic policy and the myopic policy is $(\alpha-\beta)\frac{\delta\alpha}{4\beta} - \frac{\delta\alpha}{4}(\frac{\alpha}{2\beta})^t$, where we have the following immediate result:

Lemma 3.10. *The comparison on the profit margins in the strategic policy and the myopic policy is given as follows:*

- *If* $\alpha \leq \beta$, *the profit margin in the strategic policy is always less than that in* *the myopic policy;*

- *If $\beta < \alpha < 2\beta$, there exists another critical period t_2, where $t_2 > t_1$, such that if $t < t_2$, the profit margin in the strategic policy is smaller; while if $t \geq t_2$, the profit margin in the strategic policy is larger.*

Therefore, we can see the profit margin under social interactions in the strategic policy is not necessarily larger than that in the myopic policy. Specifically, if the social interaction intensity is smaller than the unit service rate cost, the profit margin in the strategic policy is always smaller than that in the myopic policy. However, since the market size under the strategic policy becomes larger, the single period profit will be larger than that in the myopic policy if $t \geq t_1$. Many companies adopt small profits but quick turnover strategy which may not be optimal in the short term, but may be optimal in the long term under social interactions confirmed by the above result, since a smaller profit margin will lead to a large market share. However, if the social interaction intensity becomes larger, the monopolist only needs to scarify the profit margin temporarily while he eventually gets a larger profit margin and a larger market size as well as a larger single period profit in the strategic policy.

The discount factor δ is another critical factor that impacts the optimal price and service rate decisions in the strategic policy. If the discount factor δ is larger, the service rate in the strategic policy will be larger than that in the myopic policy, and the price in the strategic policy also becomes larger. The impact of δ on the profit margin depends on α and β. Specifically, if $\alpha \leq \beta$, a larger δ will drive the monopolist to scarify more profit margin in the strategic policy; while if $\beta < \alpha < 2\beta$, during the initial periods $t \leq t_2$, for a larger δ, the profit margin will become smaller, while after the initial periods, the profit margin will be become larger in the strategic policy. Therefore, in order to utilize the influence of social interactions, the monopolist should scarify more profit margin for a larger market size to increase the future profit.

Letting $N \to \infty$, the steady state in the strategic policy in terms of the service rate, price and the arrival rate, are given as the following result:

Corollary 3.3. *There exists a unique steady state in the strategic policy, where the steady state service rate, price and the arrival rate are given as*

$$\mu^{\star\star} = \mu^M + \frac{\delta\alpha}{4\beta - 2\alpha}, \; p^{\star\star} = p^M + \frac{\delta\alpha^2}{4\beta - 2\alpha}, \; \lambda^{\star\star} = \lambda^M + \frac{\delta\alpha}{4\beta - 2\alpha}.$$

Specifically,

$$\begin{aligned}
\mu^{\star\star} &= \frac{v}{2\beta} + \frac{\alpha}{2\beta}\left(\frac{v - 2\sqrt{\beta w}}{2\beta - \alpha}\right) + \frac{\delta\alpha}{4\beta - 2\alpha} \\
p^{\star\star} &= v + \alpha\left(\frac{v - 2\sqrt{\beta w}}{2\beta - \alpha}\right) - \sqrt{\beta w} + \frac{\delta\alpha^2}{4\beta - 2\alpha} \\
\lambda^{\star\star} &= \frac{v - 2\sqrt{\beta w}}{2\beta - \alpha} + \frac{\delta\alpha}{4\beta - 2\alpha}
\end{aligned}$$

The service rate path, price path and the arrival rate path monotonically converge to the corresponding steady state in the strategic policy.

Since the steady state in the strategic policy is larger than that in the myopic policy in terms of the arrival rate, we have the following three cases:

- If $\lambda_0 \leq \lambda^M$, the service rate path, price path and the arrival rate path in the myopic policy and the strategic policy monotonically converge to μ^M, p^M, λ^M and $\mu^{\star\star}$, $p^{\star\star}$, $\lambda^{\star\star}$ in an increasing order respectively;
- If $\lambda^M < \lambda_0 < \lambda^{\star\star}$, the service rate path, price path and the arrival rate path in the myopic policy monotonically converge to μ^M, p^M, λ^M in a decreasing order; while in the strategic policy, they increasingly converge to $\mu^{\star\star}$, $p^{\star\star}$, $\lambda^{\star\star}$;
- If $\lambda_0 \geq \lambda^{\star\star}$, the service rate path, price path and the arrival rate path in the myopic policy and the strategic policy monotonically converge to μ^M, p^M, λ^M and $\mu^{\star\star}$, $p^{\star\star}$, $\lambda^{\star\star}$ in a decreasing order respectively.

Therefore, compared with the myopic policy, the strategic policy always leads to a higher service rate and a higher price while inducing a higher arrival rate in order to take advantage of the social interaction effect. The steady state profit in the strategic policy is always larger than that in the myopic policy, and the gap is

$$\pi^\star(\mu^{\star\star}, p^{\star\star}, \lambda^{\star\star}) - \pi^M(\mu^M, p^M, \lambda^M) = \frac{\delta\alpha\left(2\beta\sqrt{\beta w} + 2\alpha(v - 2\sqrt{\beta w}) + \delta\alpha(\alpha - \beta)\right)}{(4\beta - 2\alpha)^2} \tag{3.8.8}$$

Besides, in steady state, the expected waiting time in the strategic policy is the same as that in the myopic policy. However, the average queue length in the strategic policy is larger.

Table 3.4 summarizes the comparison between strategic policy and myopic policy in terms of the steady state arrival rate, service rate, price, and profit.

Fig.3.8.1-3.8.4 depict the steady state arrival rate, service rate, price and profit in the myopic policy and the strategic policy with respect to α with the parameters $v = 10,\ \beta = 1,\ w = 1,\ \delta = 0.8$.

Tab. 3.4: Comparison between strategic policy and myopic policy.

Steady state arrival rate	Steady state service rate	Steady state price	Steady state profit
$\lambda^{\star\star} > \lambda^M$	$\mu^{\star\star} > \mu^M$	$p^{\star\star} > p^M$	$\pi^{\star\star} > \pi^M$

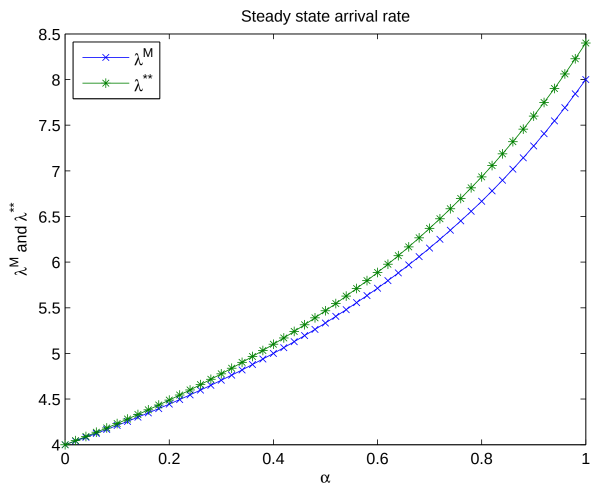

Fig. 3.8.1: Steady state arrival rate with respect to the social interaction intensity $\alpha \in [0, 1]$ under the myopic policy and the strategic policy.

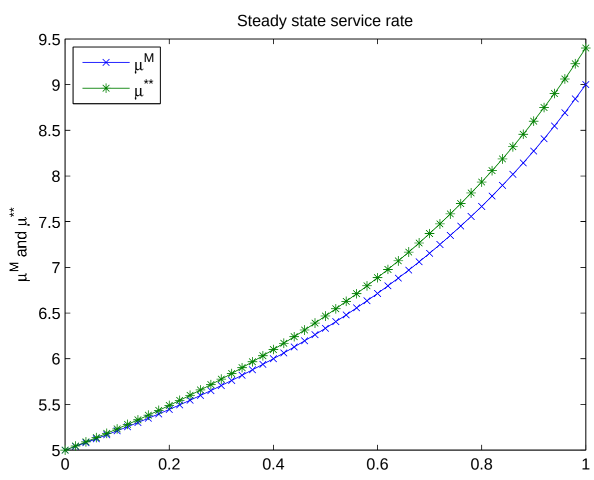

Fig. 3.8.2: Steady state service rate with respect to the social interaction intensity $\alpha \in [0, 1]$ under the myopic policy and the strategic policy.

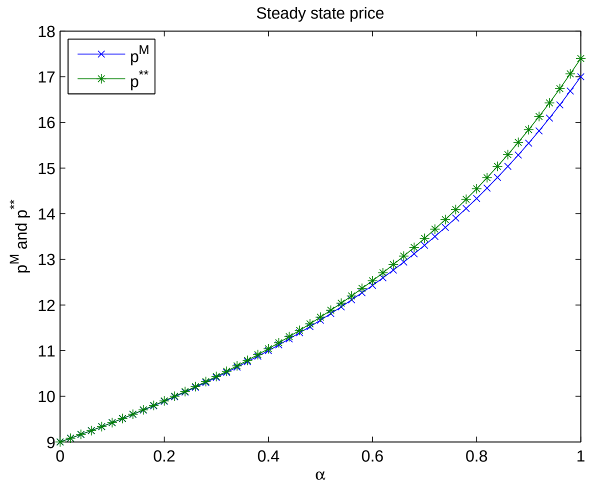

Fig. 3.8.3: Steady state price with respect to the social interaction intensity $\alpha \in [0, 1]$ under the myopic policy and the strategic policy.

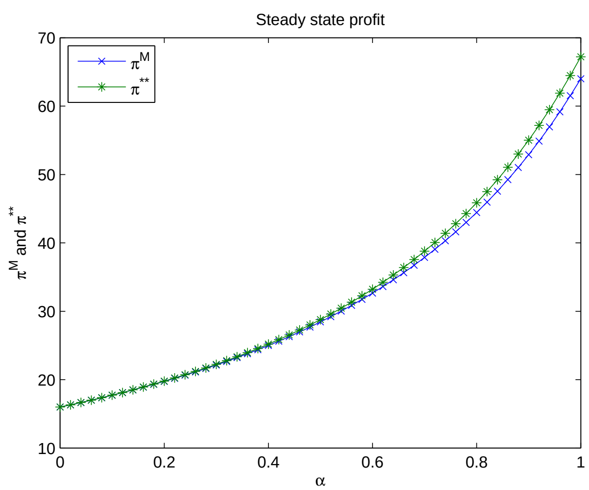

Fig. 3.8.4: Steady state profit with respect to the social interaction intensity $\alpha \in [0, 1]$ under the myopic policy and the strategic policy.

3.9 Conclusion

The significant influence of social interactions helps firms attract more customers, while it also leads to challenges in operations management, especially the capacity and price decisions. If capacity is not properly managed, more demand attracted by social interactions will increase congestion which may drive customers away, leading to potential revenue loss, a typical congestion effect. Therefore, under social interactions, how to build capacities and charge prices to balance the tradeoff between social interaction effect and congestion effect is important.

Drawn on extensive economic and marketing literature, we consider the capacity and price decisions of a monopolist when consumer purchase decisions are influenced by social interactions. Based on several dynamic settings, depending on whether managers consider social interactions in decision-making, both strategic and myopic capacity and price policies as well as the system dynamics are investigated. In each dynamic model, there exists a unique steady state arrival rate which is achieved monotonically under both strategic and myopic policies. The steady state arrival rate is always larger in the strategic policy than that in the myopic policy, i.e., the monopolist will always cover a larger market when the impact of their operational decisions is taken into consideration under social interactions. Strategic policies also lead to longer queues in the steady state. In a market with more intense social interactions, or the future profits are of concern, the strategic operational decisions should trade off more profit in the short term for a larger profit in the long term from an increased market base.

In terms of operational decisions, managers should focus on how to increase the consumer base in order to facilitate the influence of social interactions. Starting with the same initial market, managers need to sacrifice the short term profit by building a larger capacity, or charging a lower price to build up the customer base. If prices and capacities can be adjusted simultaneously, strategic policies require managers to acquire a lower profit margin to build up the customer base. Social interactions through a larger customer base will benefit firms in the long term, where a larger profit will be achieved in the steady state. However, the profit margin in the strategic policy may be always lower than that in the myopic policy, especially when social interactions are not intense. Therefore, under social interactions, our results indicate that managers who ignore long term implications of their operational decisions will consistently serve a smaller market, thereby systematically lose revenue. By adopting strategic policies, a larger customer base will help attract more potential demand, which further strengthens the influence of social interactions, i.e., firms will achieve *cumulative advantages* under social interactions.

The strategic policies driven by social interactions provide possible explanations for those interesting phenomena raised in the introduction of this chapter. The influence of social interactions may drive firms to charge a lower price in the introduction stage of their products or services. In practice, why some firms adopt seemingly suboptimal operational decisions, such as a lower price, a larger service speed and capacity, or keep a long queue may be due to the intention of building up a large customer base to better utilize the influence of social interactions. These seemingly suboptimal operational decisions may be desirable in the long term under social interactions. Similarly, in order to utilize the influence of social interactions, firms may not increase their prices or even charge a lower price facing excessive demand, because long queues or waiting lines may signal the popularity of a product or service, which will lead to more potential demand. This is shown in the results of the strategic policies. The result in steady state also explains why some firms with popular products or services can charge a higher price or obtain a higher profit margin than their competitors even if similar or the same products or services are offered: companies with reputation may enjoy cumulative advantages from a larger customer base.

The strategic policies indicate under social interactions, facing a small initial market, appropriate operational decisions should induce a steadily expanding market base, through smooth price adjustments or gradual capacity expansions. Drastic change of operational decisions may lead to sudden sales explosion or contraction, which may significantly impact the future demand under social interactions. The influence of social interactions may explain why some firms suffer from a significant demand drop after increasing their prices, which eventually leads to a contracted market share, or even bankruptcy. The increased price will lower the current sales volume, which will decrease the potential demand under social interactions whose influence will be further reduced from even lower demand, i.e., *cumulative disadvantages* may lead to a contracted market. The influence of social interactions may also explain why firms of the same industry adopt different operational policies in terms of price, capacity, or profit margin. Probably, products or services of some companies can stir up more customer involvement and interactions to help brand dissemination, where firms can charge higher prices compared with their competitors. Through operational decisions, social interactions may also drive some firms to adopt the market strategy with a small profit margin but a large market, while a high profit margin with a small market for other companies.

4. DYNAMIC SERVICE EFFORT DECISION UNDER SOCIAL INTERACTIONS

4.1 Abstract

Consumer purchase decisions are highly influenced by others through social interactions, where consumer satisfaction plays an important role. Satisfaction is primarily determined by pre-purchase expectations and post-purchase experiences which are both affected by resource investment in service quality during product deliveries and service encounters, termed as *service effort decision*. Under social interactions, consumer satisfaction significantly influences purchase decisions of both existing and potential consumers. How service effort decisions will be impacted by social interactions needs to be better understood. Appropriate service effort decisions help firms enhance consumer satisfaction and improve performance through properly managed expectations and experiences in product deliveries and service encounters. In this chapter, we propose a dynamic service effort decision model for a profit maximizing firm, which offers frequently purchased product or service to heterogeneous, backward-looking and adaptive consumers under social interactions, particularly through word-of-mouth communication.

Through repeated interactions, both expectations affected by past service encounters and experiences determined by service effort, determine consumer satisfaction in service delivery. Under optimal service effort policies, consumer overall experiences monotonically converge to a unique steady state level. Social interactions always drive firms to exert a high level of service effort which enhances consumer satisfaction. Service expectations also impact service effort decisions; if consumers rely more on overall experiences from past service encounters in service expectations, a higher level of effort should be exerted to offer a higher level of service quality. If social interactions are satisfaction-dependent, a constant service effort policy may be optimal, especially when consumers have high initial service expectations. Being aware of social interactions and consumer adaptations in service expectations as well as their interplay may contribute to better managed consumer expectations, experiences and satisfaction in service delivery so as to increase profitability.

4.2 Introduction

Social interactions significantly influence consumer purchase decisions. From a managerial perspective, how service should be offered in both product deliveries and service encounters to properly manage *expectations* and *experiences* and thus enhance *consumer satisfaction* becomes critical in operations to expand market share and increase profitability under social interactions.

Nowadays, firms can take various marketing strategies and campaigns to stimulate demand and promote popularity of their products or services, through creation of buzz and forums, providing online customer reviews, etc., to utilize the influence of social interactions. As consumers become overloaded, they have become increasingly skeptical about traditional company-driven advertising and marketing and increasingly prefer to make purchase decisions largely independent of what companies tell them (Bughin et al., 2010), in other words, they become more reliant on consumer-generated information, such as recommendations, opinions and choices of others. Social interactions, if properly utilized through operations, may help firms easily establish brand reputations and recognitions. However, under social interactions, firms may also suffer from negative publicity substantially due to inappropriate operations, where some negligible unfavorable information may lead to sudden demand drop, reputation damage, and even ruination of their fame and drag firms into bankruptcy[1].

The influence of social interactions, especially WOM communication, is closely related to consumer satisfaction in service or product delivery. Under social interactions, *satisfied consumers* from pleasant purchase experiences will help attract potential consumers to make the same choice, termed as *the positive social interaction effect*; while *dissatisfied consumers* due to disappointed purchase experiences may discourage potential consumers purchasing from the same firm, termed as *the negative social interaction effect*. Consumer satisfaction and dis-satisfaction substantially impact firm performance. Under social interactions, satisfied consumers may lead to more repurchase decisions and more recommendations that help draw even more demand. Research has shown that for contact centers, satisfied consumers will be more likely to make contact again by as much as 149%, and they are more likely to recommend the company to others by as much as 180%[2]. Consumer dissatisfaction may lead to significant revenue drop. Improving consumer satisfaction to build brand reputation and sales is critical, even for big companies, such as McDonald's[3]. It has been found that the spread of positive and negative messages from satisfied and dissatisfied consumers can increase firm market share by as much as

[1] http://online.wsj.com/article/SB10001424127887323706704578229362923520102.html
[2] http://online.wsj.com/article/PR-CO-20130502-909140.html
[3] http://online.wsj.com/article/SB10001424127887324010704578414901710175648.html

10% or reduce it by 20% over a two-year period (Bughin et al., 2010).

Consumer satisfaction is mainly determined by *pre-purchase expectations* and *post-purchase experiences*, which are the two critical factors that impact consumer repurchase decisions and social interaction activities. Lower expectations can be met easily from past experiences leading consumers to be satisfied with their purchase decisions[4]; while higher expectations may always be hard to satisfy which will drive consumers to become dissatisfied with their purchase experiences[5]. Properly managed expectations in operations can help improve consumer satisfaction significantly, especially through lowering consumer expectations. For example, letting patients know that their messages may receive a delayed response when they start sending messages to doctors, increases the patients' satisfaction substantially[6], since expectations have been properly managed. Firms can adopt the "under-promise, over-deliver" (UPOD) strategy to serve their customers, which may also help engender consumer loyalties significantly, due to reduced consumer expectations[7].

Consumer purchase experiences are mainly affected by operations, especially how services are delivered during purchasing processes or service encounters, the *service effort decision*. As a broad term, service can refer to a specific service consumed by customers and the service associated with product consumption. Service effort directly determines the *service quality* and *service value* during service encounters. As a broad term, service quality can be measured by the attribute of a specific service offered by a firm, such as the waiting time in call centers and fast food restaurants, the connection speed of internet services, the signal strength and coverage of telecommunication services, the delivery speed of transportation services, the accuracy and effectiveness of diagnosis and expert services, and so on. For the service associated with product consumption, service quality may be measured by the effort of sales representatives, shopping environment, supporting service after sales, product availability (service level) of a retailer (Gaur and Park, 2007), the lead time of an order from a make-to-order firm, or even the product quality itself, etc. Due to service characteristics, such as intangibility, heterogeneity, and inseparability (Parasuraman et al., 1985), service deliveries involve more consumer contacts, and service quality may highly correlate with consumer expectations and experiences, which is more consumer-dependent (Parasuraman et al., 1985; Harvey, 1998). Under social interactions, as a determinant factor of service quality, service effort decisions become critical in operations, which influences consumer satisfaction substantially in service transactions.

[4] http://online.wsj.com/article/SB124864862273182247.html

[5] http://www.nytimes.com/2000/03/26/automobiles/expensive-sport-utilities-fall-short-of-expectations.html

[6] http://www.amednews.com/article/20130422/business/130429969/5/

[7] http://online.wsj.com/article/SB10001424052702304203304576447823427183788.html

High quality service may significantly enhance consumer service experiences, thus satisfaction; while poor quality of service may substantially harm consumer purchase experiences leading to dis-satisfaction. Although always desired, providing a high level of service quality may require more effort and resource investment, which may increase operational cost and reduce profit margins. Service effort decisions also impact consumer expectations, especially for frequently purchased products or services due to repeated interactions between consumers and firms. Consumers may have previous experiences from past service encounters, which will impact expectations when making repurchase decisions due to *reference effect* or *adaptation* in expectations formation (Kahneman and Tversky, 1979). Service effort impacts both experiences and expectations, and thus satisfaction of both *existing customers* and *potential customers* under social interactions. Therefore, how service should be delivered with appropriate and acceptable quality to enhance experiences and meet expectations and thus effectively manage consumer satisfaction through service effort decisions becomes a critical challenge for managers to tackle to better leverage the influence of social interactions in profitability and market expansion.

In this chapter, based on the *service quality gap* model (Parasuraman et al., 1985), we investigate service effort decisions for a profit maximizing firm under social interactions. Our focus is to investigate how social interactions would impact service effort decisions to better manage expectations, experiences and consumer satisfaction in profit optimization. Specifically, (1) what is the impact of social interactions on consumer expectations, experiences and satisfaction? (2) how would consumer expectations impact service effort decisions? (3) what is the impact of social interactions on service effort decisions? (4) should a constant level of effort be maintained to offer a consistent quality of service or vary service effort in profit optimization? Based on an infinite-horizon dynamic service effort decision model, under several dynamic settings, optimal service effort decisions and their structural properties provide answers to the above questions.

The remaining chapter is organized as follows. Section 4.3 briefly reviews several related studies on the influence of social interactions in consumer purchase decisions. A dynamic service effort management model under social interactions is built in Section 4.4. Section 4.5 characterizes the optimal service effort decision in the benchmark model where consumer expectations are independent of past experiences. We investigate the optimal service effort policy where consumers are adaptively in expectation formation in Section 4.6. The model is extended in Section 4.7, where consumer purchase volumes are influenced by satisfaction. We generalize the model to the situation where social interactions are dependent on consumer satisfaction in Section 4.8. Discussion and conclusions as well as future research directions are provided in Section 4.9.

4.3 Literature Review

The primary research question in this study is to investigate *how social interactions would impact service effort decisions in product and service deliveries to better manage consumer expectations, experiences, and satisfaction in profit optimization.* We first focus on studies on the influence of social interactions, especially the WOM in consumer purchase behaviors and how firms utilize WOM to promote sales and drive profitability through operational decisions. Manski (2000) and Hartmann et al. (2008) offer brief reviews on the role of social interactions in consumer purchase decisions. We then briefly review several studies on operational decisions in terms of service effort decisions in service and product deliveries.

Consumer purchasing behaviors influenced by social interactions have been widely studied both empirically and theoretically. As a typical form of social interaction, the role of WOM communication has been extensively studied. Incorporating WOM into the diffusion process, Bass (1969) developed one of the most important marketing models, the *Bass diffusion model* to study the diffusion process of a new product. Dodson and Muller (1978) build a new product diffusion model with interactions between adopters and non-adopters as well as the impact of advertising. Mahajan et al. (1984) discuss the new product introduction strategy and the optimal advertising timing policy with positive and negative WOM using a diffusion model. Similar study has been conducted by Kalish and Lilien (1986). Due to social interactions, the value of the consumer to firms is beyond what he/she has bought. An existing consumer may attract or discourage potential consumers to buy the same product through social influences, which leads to the value of a consumer including both her purchasing value and her influence value (Ho et al., 2012).

Several studies have focused on the measurement and the effectiveness of WOM. To measure WOM activity through the *recommend intention metric*, Aksoy et al. (2011) provides a longitudinal study on the relationship between recommend intention and the adoption of a new-to-market service brand extension. The findings indicate the recommending consumers are more recent adopters of the service and are in more frequent contact with potential consumers. Narayan et al. (2011) investigate the effects of peer influence in consumer product choices with multiattribute. In the model, consumers who are uncertain about their attribute preferences update their attribute preferences in a Bayesian mechanism incorporating peer influence. Gu et al. (2011) distinguish the source of WOM, namely the internal WOM which is provided by the retailers and the external WOM otherwise. Using the sales data of high-involvement product like a camera from Amazon.com, the result indicates external WOM sources have a significant impact on sales. The empirical study by Godes and Mayzlin (2009) indicates WOM created between less loyal consumers and their acquaintances is more effective to drive sales. The impact of WOM is even more effective than social learning where individuals directly observe the choices of others. A similar result is concluded by Celen et al. (2010) using a laboratory experiment, where subjects appear to be more willing to follow the advice of their predecessor than to copy their actions.

Interpersonal communications have long been recognized as an influential information source for consumers to make purchase decisions. Increasing popularity of various online social media has substantially facilitated social interactions among consumers. Iyengar et al. (2011) study the role of opinion leadership and social contagion within social networks in new product adoption. The results indicate the amount of contagion is moderated by both the recipient's perception of their opinion leadership and the volume of product usage. Sonnier et al. (2011) investigate the effect of online communications on firm sales, including the positive, negative and neutral effects. Consumer online discussions not only impact firm sales but also impact stock market performance, such as the study by Tirunillai and Tellis (2012), where user-generated-content (UGC) and its volume have a significant positive effect on abnormal returns and trading volumes of firm stock market shares. There is a growing evidence that consumer perceptions are significantly influenced by internet-based opinions before making purchase decisions, which significantly impact firm profit and consumer surplus (Dellarocas, 2006). Community participation also impacts consumer online buying and selling behaviors (Algesheimer et al., 2010). Through online communications, informed consumers share price information to attract uninformed consumers to form group buying through social interactions, where quantity discounts can be enjoyed (Jing and Xie, 2011).

Envisioning the influence of social interactions in consumer purchase behavior, several studies have focused on firm strategies to increase sales and profitability. Mayzlin (2006) analyzes the equilibrium strategy for a firm to provide consumer reviews, where consumers can get information about product quality from evaluations of other consumers. Social interactions on consumer purchase behavior are correlated with satisfaction which impacts consumer retention. Anderson and Sullivan (1993) find that satisfaction is best specified as a function of perceived quality and "disconfirmation"-the extent to which perceived quality fails to match pre-purchase expectations. There exists an asymmetric effect on consumer satisfaction and retention. Specifically, the perceived quality which falls short of expectations has a greater impact on satisfaction and repurchase intentions than quality which exceeds expectations. Ho et al. (2006) model consumer purchase rate as Poisson events with the rates dependent on satisfaction of the most recent purchase encounters. Bolton (1998) studies the role of consumer satisfaction on the retention in the long-run supplier-consumer relationship. In the model, consumers update their subjective expected value of the relationship according to an anchoring and adjustment process, where the cumulative satisfaction serves as an anchor which is updated with new service experiences. Bolton et al. (2006) investigate the effect of service experiences on firm renewal decisions with suppliers, and the result indicates, recent experiences are weighted more than earlier experiences.

Since consumer expectations are crucial to satisfaction and retention, firms begin to focus on the management of consumer expectations, especially how many resources and how much effort should be invested in service delivery. Always delighting the consumer may raise consumer expectations, making them more difficult to be satisfied in the next purchase cycle, which hurts the firm in the long run (Rust and Oliver, 2000). Ho and Zheng (2004) investigate how a firm chooses a delivery time commitment to influence its consumer expectations and delivery quality in order to maximize its market share. Kopalle and Lehmann (2006) investigate optimal advertised quality, actual quality, and price for a firm to enter into a market. Based on a two-period model, advertised quality influences expectations, and consumer satisfaction are determined by the gap between actual quality and expectations. Rust et al. (1999) model consumer expected service quality that they will experience as a distribution based on their cumulative experiences. Aflaki and Popescu (2012) build a dynamic model of the firm-client relationship to study the optimal service level policy. In their model, a representative consumer expectation is randomly distributed and the retention function is a probability depending on consumer past experiences.

From the above studies, we can see there are few studies directly addressing how social interactions would impact operational decisions in service delivery and the dynamics of consumer expectations, experiences and satisfaction. The current study aims to contribute to the literature by building a dynamic service effort decision model to investigate the impact of social interactions on operations and the related dynamics in consumer purchase decisions.

4.4 A Service Effort Decision Model Under Social Interactions

4.4.1 Model Settings

A profit-maximizing firm which offers frequently purchased product/service in the consumer market decides how many resources should be devoted to service delivery, termed the *service effort* decision, where consumer purchase decisions are influenced by social interactions, typically WOM communication. We assume the service quality or service value is primarily determined by the level of service effort. For consumer-intensive services, such as health care, legal and financial consulting and personal care services, the service quality is positively correlated with service durations (Anand et al., 2011), i.e., the longer service time, the higher service quality.

In this chapter, the service quality may be associated with either a particular

Tab. 4.1: Key variables and notations.

Variables	Notations	Variables	Notations
Service effort level	q	Service expectation distribution	$F(q^E, r)$
Consumer service expectation	Q^E	Effective demand	d
Realized service expectation	q^E	Positive social interaction intensity	$\kappa(q, r)$
Consumer service experience	q^A	Negative social interaction intensity	$\tau(q, r)$
Overall experience	r	Service price	p
Expected service value	U^E	Discount factor	δ
Realized expected service value	u^E	Memory factor	γ
Experienced service value	u	Profit function	$\pi(q, r)$

service or a product. For ease of exposition, we assume a frequently purchased service is offered by the firm. Service effort impacts service quality and service value perceived by consumers. High level of service quality may always be costly from the firm perspective, since a high level of service effort should be exerted. Thus, we consider *service effort and service quality are positively correlated* in this chapter as an implicit assumption. A high level of service effort will lead to a high quality of service which will enhance consumer service experiences, which may increase consumer satisfaction. However, the operational cost of service delivery may increase substantially if the service effort level becomes high enough. Therefore, managers need to balance the tradeoff between high quality of service and high service effort cost. Key notations used in this chapter are listed in Table 4.1:

At the beginning of each period t, before purchasing the service, potential consumers form service expectations, where they have expected value from service consumption[8]. Consumers are *heterogeneous* in expectations, i.e., their service expectations may be different from one another, which is captured by a random variable U_t^E and u_t^E is the realization of the service value expectation for a representative consumer. For frequently purchased service, due to repeated interactions, consumer purchase decisions are influenced by expectations and experiences. Generally, there are two types of consumers in terms of purchase decision making, namely *forward-looking* and *backward-looking*. Forward-looking consumers make purchase decisions based on expectations and reservations; while backward-looking ones make the consumption choice from expectations and past experiences.

We focus on the backward-looking consumers in the repeated interaction setting[9]. Particularly, at the end of period t, a consumer who has purchased the service forms experienced service value u_t. The experienced service value may also be different among consumers, due to their heterogeneity in perceptions, expectations, or

[8] Consumer service expectations may also refer to service attributes, such as the expected waiting time, lead time, etc.

[9] The model can also be interpreted with forward-looking consumers with slightly changes, where their experiences may impact their reservations.

the service transactions. In this chapter, we assume service quality is the primary factor that determines consumer experienced service value. Therefore, we mainly consider the heterogeneity of consumers in terms of their expected service values, while the experienced service values are kept the same among all consumers, which is determined by service effort. The model can be extended to situations where consumers are heterogeneous in terms of their experienced service value only or both the expected and experienced service value.

As discussed before, expectation before purchasing and experience after consumption jointly determine consumer satisfaction, which impacts whether a consumer is satisfied or dissatisfied from service experience due to the backward-looking behavior. A satisfied consumer whose service experience is no less than the expectation will probably make repurchase decisions from the same firm; while a dissatisfied one whose experience does not meet expectation may terminate the transaction immediately or may not even come back for future consumption. Consumers are also influenced by their backward-looking inclinations in expectation formation, where they may *adaptively* change and update service expectations from overall experiences through past service encounters.

In each period, a consumer who decides to purchase the service, is either as an *existing consumer* who has purchased the service in the previous period, or a *potential consumer* (or new consumer) who does not have transaction experiences with the firm before. Due to positive social interaction effect, potential consumers will be attracted to purchase the service through satisfied consumers directly or indirectly, such as recommendations through WOM, on-line consumer reviews, or even observations from previous sales. While under negative social interaction effect, potential consumers may be discouraged from purchasing the service due to interactions with dissatisfied consumers. Through social interactions, service expectations of potential consumers may be significantly influenced by existing consumers, although they have had no transaction experiences with the firm before. In this chapter, we assume *potential consumers have the same expectations as existing consumers in terms of expected service value distributions.*

Therefore, under social interactions, in each period, the total effective demand of the firm which will be served is composed of those existing consumers who are satisfied and the attracted potential consumers. As discussed before, consumer expectations may be influenced by past experiences due to backward-looking behavior. Thus, we consider consumers are *reference dependent* in expectation formation. Particularly, we assume *the distribution of consumer expected service value depends on previous service experiences* with the firm, termed as the *overall experience* (or overall perception), which will be discussed in the following section in detail. Due to repeated interactions and *reference effect* (Popescu and Wu, 2007), the reference service value will be updated and adjusted frequently in time, especially due to *anchoring and adjustment* inclination in consumer decision-making.

The objective of the firm is to make appropriate service effort decisions to better manage consumer expectations, experiences and satisfaction with the purpose of maximizing the long-run expected profit under social interactions. The model is termed *service effort decision model* or SED model for simplicity. We focus on the structural properties of the optimal service effort policy, such as the monotonicity of the effort level, the existence of the steady state service effort levels, and the impact of various behavioral factors on the service effort decisions. The detailed model is discussed in the following section.

4.4.2 Service Effort Decision Under Social Interactions

4.4.2.1 Consumer Expectation, Experience and Satisfaction

In this section, we discuss the detailed formulation of the above SED model. As discussed in the above section, service value and service quality are usually positively correlated; in the following section, we use service quality and service value interchangeably without confusion. Consumer experienced service value is primarily determined by service quality, where the latter is mainly impacted by service effort decision. Since the more service effort, the higher service quality and larger service value, we assume *there exists a one-to-one correspondence between service effort and service value*, formulated as $u = \phi(q)$. Similarly, consumer expected service value and service expectation are also positively correlated as $u^E = \varphi(q^E)$; consumer service experiences are primarily determined by experienced service value as $q^A = \psi(u)$. For ease of analysis, in the following section, we assume the simple relationships $u = q$, $u^E = q^E$, $q^A = u$, which implies $q^A = q$, i.e., consumer service experiences are determined by service effort. The model can be easily generalized to more complex functional forms as long as $\phi(q)$, $\varphi(q^E)$, $\psi(u)$ are increasing functions, where all the key results remain unaffected.

As discussed before, consumer service expectations are formed based on overall experiences r_t during previous service encounters from the initial period until period $t-1$. Since we only consider consumer heterogeneity in pre-purchase service expectations, we assume overall experience *in each period is the same for all consumers.* Consumers form initial service experience r_0 which may be influenced by firm advertising campaigns, reputation, industry standards and other factors, such as the experience of substitutable services from competing firms, etc. The heterogeneity of consumers in terms of service expectations is modeled as follows. In each period t, the distribution of the service expectation of a representative consumer or the total population of all consumers is a random variable $Q_t^E \in [\underline{q}, \bar{q}]$, $\underline{q} \leq q_L$, $\bar{q} \geq q_H$

whose distribution is anchored at the overall experience r_t, with the cumulative distribution function $F(q_t^E, r_t) = \Pr(Q_t^E \leq q_t^E | r_t)$[10]. Intuitively, due to the reference effect, the average service expectation will become larger if the overall experience is higher, due to consumer adaptations. Therefore, we assume the distribution function $F(\cdot, r)$ satisfies the *first-order stochastic dominance (FSD)* property. Denote $q^E(r) = \int_{\underline{q}}^{\bar{q}} q^E \mathrm{d}F(q^E, r)$ as the mean of service expectation. The FSD property indicates, if $r' \geq r$, then $F(q^E, r') \leq F(q^E, r)$, which indicates $q^E(r') \geq q^E(r)$.

Consumer satisfaction is determined by expectation and experience. Since consumer service experience q_t^A is equal to firm service effort q_t, after consuming the service in period t, the probability of an individual consumer who is satisfied with the service experience is $F(q_t, r_t) = \Pr(Q_t^E \leq q_t^A | r_t)$, while dissatisfied with probability $1 - F(q_t, r_t)$. Therefore, service effort determines consumer satisfaction. From the whole population perspective, $F(q_t, r_t)$ is the proportion of satisfied consumers, and $1 - F(q_t, r_t)$ is the proportion of dissatisfied consumers. In the following section, we refer to $F(q, r)$ as the probability and proportion interchangeably without ambiguity. Denote F_1 and F_2 as the first order derivative of $F(q, r)$ in terms of q and r respectively. The corresponding second order derivatives and cross partial derivatives are denoted as F_{11}, F_{22} and F_{12}. Besides the FSD property of the random service expectation, we assume the distribution function $F(q, r)$ has the following additional properties:

- *Diminishing reference effect in consumer satisfaction:* $F(q, r)$ is decreasing and convex in r, i.e., $F_2(q, r) = \frac{\partial F(q,r)}{\partial r} < 0$, $F_{22}(q, r) = \frac{\partial^2 F(q,r)}{\partial r^2} \geq 0$. The assumption indicates the probability of a consumer whose service expectation is no more than q is decreasing and convex in r, in the sense that, a higher overall experience will induce more consumers to form a larger service expectation in the stochastic sense. Therefore, given the actual service effort, the proportion of satisfied consumers will decrease if overall experience increases. However, the impact of overall experience in consumer satisfaction is diminishing.

- *Increasing service experience and service effort effect in consumer satisfaction:* $F(q, r)$ is supermodular in (q, r), i.e., $F_{12}(q, r) \geq 0$. The supermodular property of $F(q, r)$ indicates if $q_2 \geq q_1$ and $r_2 \geq r_1$, we have $F(q_2, r_2) - F(q_1, r_2) \geq F(q_2, r_1) - F(q_1, r_1)$, i.e., given two service effort levels, the difference betweem the proportion of satisfied consumers increases in overall service reference.

- $F(q, q)$ is increasing and concave in q, i.e., when service experience and overall experience are identical, the proportion of satisfied consumers is increasing and concave. We denote $f(q) = \frac{\partial F(q,q)}{\partial q} = F_1(q, r) + F_2(q, r)|_{r=q} \geq 0$ with $f'(q) \leq 0$.

[10] Due to the assumed one-to-one relationship, Q_t^E and U_t^E have the same distribution. The domain of Q_t^E may depend on r_t. For ease of analysis, we do not consider the dependance in this chapter.

We discuss how consumers update overall experiences from past service encounters. Behavioral theories have demonstrated that consumers update and adjust their perceptions gradually based on their experiences due to the anchoring and adjustment inclination. The widely adopted Bayesian updating mechanism in consumer decision-making also confirms that consumers adaptively change and adjust their perceptions gradually. Therefore, we assume consumers gradually update their overall experiences from their past purchase decisions according to the widely used *exponential smoothing rule*, as

$$r_{t+1} = \gamma r_t + (1-\gamma) q_t^A = \gamma r_t + (1-\gamma) q_t \tag{4.4.1}$$

where $\gamma \in [0,1]$ denoted as *the memory factor* depicts how important the most recent service experience is in the overall experience, i.e., consumers overall experience r_{t+1} is a weighted sum of the previous overall experience r_t and the recent service experience determined by firm effort q_t. If $\gamma = 0$, consumers are *memoryless*, i.e., they only remember their most recent service experiences $r_{t+1} = q_t$; while if $\gamma = 1$, consumers are *memorable* and they always perceive the overall experiences based on their initial perceptions, i.e., $r_t = r_0$ for $t \geq 1$. We assume the initial overall experience is in the range $r_0 \in [q_L, q_H]$ and since $q_t \in [q_L, q_H]$, the overall experience in each period is always bounded as $r_t \in [q_L, q_H]$ for all $t \geq 0$.

4.4.2.2 Demand Dynamics Under Social Interactions

Through social interactions, consumer satisfaction from service experiences influences repurchase decisions of existing consumers and service choices of potential consumers. Therefore, the total potential number of consumers that demand a service in period $t+1$ denoted as A_{t+1} depends on the potential demand A_t, the effective demand served d_t, the proportion of satisfied consumers $F(q_t, r_t)$ as well as the proportion of dissatisfied consumers $1 - F(q_t, r_t)$. We assume the potential demand dynamics under social interactions as the following general form

$$A_{t+1} = G\left(A_t, d_t, F(q_t, r_t)\right) \tag{4.4.2}$$

which is increasing in A_t, d_t and $F(q_t, r_t)$, while decreasing in $1 - F(q_t, r_t)$. In the sense that, the larger potential demand and existing consumer base, the more potential demand; the larger proportion of satisfied consumers, the more potential demand; while the larger proportion of dissatisfied consumers, the less potential demand. The implicit assumption in the above demand dynamics is that we consider *an open market and there is no constraint on the market size.*

In each period t, the effective demand depends on the potential demand and the fixed service price. In the following section, the linear demand function is assumed as an example

$$d_t = A_t - \phi p \tag{4.4.3}$$

where ϕ captures the price sensitivity. We assume $d_t \geq 0$ and $d_t \leq A_t$ for any $t \geq 0$ in this chapter. For tractability, we assume a linear form of the potential demand dynamics under social interactions as

$$\begin{aligned} A_{t+1} &= A_t + d_t \left(\kappa(q_t, r_t) F(q_t, r_t) - \tau(q_t, r_t)(1 - F(q_t, r_t)) \right) \\ &= A_t + d_t \left((\kappa(q_t, r_t) + \tau(q_t, r_t)) F(q_t, r_t) - \tau(q_t, r_t) \right) \end{aligned} \tag{4.4.4}$$

where $\kappa(q_t, r_t) \geq 0$ is defined as *the positive social interaction intensity* and $\tau(q_t, r_t) \geq 0$ is termed as *the negative social interaction intensity*. We denote $S(q_t, r_t) = (\kappa(q_t, r_t) + \tau(q_t, r_t))F(q_t, r_t) - \tau(q_t, r_t)$. Social interaction intensities may depend on overall service experience and service effort. We will discuss the dependance in the following section. For the base model, we assume social interaction intensities are constant as κ and τ respectively.

If $\kappa \geq \tau$, we have a stronger positive social interaction effect, i.e., a satisfied consumer can attract more potential demand than the loss from a dissatisfied one; while if $\kappa < \tau$, we have a stronger negative social interaction effect, i.e., dissatisfied consumers may discourage more potential consumers than those drawn by satisfied ones. The strength of social interaction can be empirically measured from real data. For example, the survey conducted by McKinsey Quarterly suggests that in the mobile-phone market, the pass-on rates for key positive and negative messages can increase a company's market share by as much as 10 percent or reduce it by 20 percent over a two-year period (Bughin et al., 2010). The empirical study by Anderson (1998) suggests that WOM from dissatisfied consumers is greater than that from satisfied ones.

Given the initial potential demand size A_0 which may depend on the reputation of the firm, the condition in consumer market, the service characteristics, etc., the initial sales quantity is $d_0 = A_0 - \phi p$. In period t, the potential demand A_t and the effective demand d_t can be formulated as

$$\begin{aligned} d_t &= A_t - \phi p = A_{t-1} + d_{t-1} S(q_{t-1}, r_{t-1}) - \phi p = d_{t-1} \left(1 + S(q_{t-1}, r_{t-1})\right) \\ &= \left(1 + S(q_{t-1}, r_{t-1})\right) (A_{t-1} - \phi p) = d_{t-2} \left(1 + S(q_{t-1}, r_{t-1})\right) \left(1 + S(q_{t-2}, r_{t-2})\right) \\ &= \cdots = d_0 \prod_{\tau=0}^{t-1} \left(1 + S(q_\tau, r_\tau)\right) \end{aligned}$$

The above demand dynamics indicate, if the proportion of satisfied consumers is small, such that $\frac{\tau-1}{\kappa+\tau} \leq F(q_t, r_t) \leq \frac{\tau}{\kappa+\tau}$ for all $t \geq 0$, the firm will lose consumers gradually and may have to exit the market eventually, since $0 \leq 1 + S(q_t, r_t) \leq 1$; while if $F(q_t, r_t) \geq \frac{\tau}{\kappa+\tau}$ for all $t \geq 0$, the firm will attract more and more potential demand gradually in time and expand its market share to cover the whole market eventually. Therefore, the proportion of satisfied consumers plays an important role in demand dynamics. Although, the model is stylized and simple, it indeed captures the real situation in practice to some extent. In the following section, we make the following technical assumptions

$$(\kappa+\tau)F(q_L, q_H) - \tau + 1 \geq 0, \ \lim_{t\to\infty} \delta^t \left((\kappa+\tau)F(q_H, q_L) - \tau + 1\right)^t = 0$$

where δ is *the discount factor*. The first condition indicates, for any $q \in [q_L, q_H]$, and $r \in [q_L, q_H]$, we have $0 \leq 1 + S(q, r)$, even if the service effort is the lowest and overall consumer experience is the highest, where the proportion of satisfied consumers is the lowest as $F(q_L, q_H)$. The second condition is to guarantee the long-run discounted profit is bounded, since for any $q \in [q_L, q_H]$, and $r \in [q_L, q_H]$, the sequence of demand (also the single period profit) after discounting is finite and convergent, so that the long-run discounted profit will be bounded and convergent. A sufficient condition for the second assumption to be satisfied is $\delta(\kappa+1) \leq \rho$, where $\rho \in (0, 1)$, which will be used in the analysis in the following section.

4.4.2.3 Service Effort Decision Under Social Interactions

The time-line of the model is as the following: at the beginning of each period t, the total number of served consumers is d_t; consumer overall experience is updated as r_t; consumers form service expectations Q_t^E; the firm decides the service effort q_t; consumer satisfaction is realized; the period changes to $t + 1$[11].

The service effort cost is assumed to be increasing and convex in the effort level, denoted as $C(q_t)$. The profit in each period is denoted as $\Pi(q_t, r_t, d_t) = k(q_t, r_t)(p - C(q_t))d_t$, where $k(q_t, r_t)$ denotes the purchase volume for an individual consumer. For an initial reference quality r_0 and potential demand size A_0, the long-run discounted profit maximizing problem is formulated as the following dynamic

[11] As discussed in the previous section, we realize that consumer evaluation of the service quality may be biased due to their cognitive and affective factors, such as their preconceptions. For example, built on Categorization Theory and Perception Distortion Theory, Iglesias (2004) finds that there exists strong effect of preconceptions about the service category on the perceptions of quality during the service encounter. In this chapter, we do not consider this effect in our model.

programming model:

$$
\begin{aligned}
V(r_0, A_0) \quad &= \quad \max_{q_t \in [q_L, q_H],} \sum_{t=0}^{\infty} \delta^t \Pi(q_t, r_t, d_t) \\
s.t. \quad & \quad r_t = \gamma r_{t-1} + (1-\gamma) q_{t-1},\ t \geq 1 \\
& \quad A_t = A_{t-1} + d_{t-1}\left((\kappa+\tau)F(q_{t-1}, r_{t-1}) - \tau\right),\ t \geq 1 \\
& \quad d_t = A_t - \phi p,\ t \geq 0
\end{aligned}
\tag{4.4.5}
$$

Using the overall experience and the effective demand as the state variables in each period, the above profit maximizing problem can be reformulated as a two-dimensional infinite-horizon dynamic programming problem

$$
V(r, d) = \max_{q \in [q_L, q_H]} \Pi(q, r, d) + \delta V\left(\gamma r + (1-\gamma) q, d\left((\kappa+\tau)F(q, r) - \tau + 1\right)\right) \tag{4.4.6}
$$

and $V(r, d)$ is the unique bounded solution, since the single period profit is finite based on the assumptions on the demand dynamics. Since the single period profit function is separable in terms of the number of consumers, i.e., $\Pi(q, r, d) = d\pi(q, r)$, where $\pi(q, r) = k(q, r)(p - C(q))$ is denoted as the expected profit from an individual consumer, the effective demand size does not impact the optimal decisions. Therefore, the initial demand is normalized as $d_0 = 1$, and the above model can be reduced as

$$
V(r) = \max_{q \in [q_L, q_H]} \pi(q, r) + \delta\left((\kappa+\tau)F(q, r) - \tau + 1\right) V\left(\gamma r + (1-\gamma) q\right) \tag{4.4.7}
$$

Therefore, the model developed in this chapter can also be interpreted with a representative consumer, where the value function $V(r)$ captures the long-run discounted life-time value of each consumer under social interactions.

Compared with previous studies, especially the model developed in Aflaki and Popescu (2012), we can see our model is more general. Existing models lack consideration of either the influence of social interactions in consumer purchased decisions, or the reference effect in consumer expectations. By incorporating social interactions and reference effect in consumer purchase behaviors, our model captures the reality in consumer market to a great extent.

4.5 Service Effort Decision with Myopic Consumers Under Social Interactions

In this section, we consider the service effort decision in the situation where the single period profit only depends on consumer service experiences, i.e., the service effort, and consumer service expectations at the beginning of each period are independently and identically distributed without reference effect. The model is termed *the benchmark model*. Consumers are *myopic*, since their service expectations are independent of their past overall experiences and only the most recent service experience impacts satisfaction. Consumers are still backward-looking and influenced by social interactions in purchase decision-making. The benchmark model is also applicable for those services which are not frequently purchased, where consumers generally interact with firms for limited times, such as in non-contractual settings.

Therefore, at the beginning of each period, consumer service expectation Q^E is a random variable with distribution $F(q^E) = \Pr(Q^E \leq q^E)$ in the domain $[\underline{q}, \bar{q}]$. Each customer will purchase a constant unit of service and we also assume the single period profit function $\pi(q)$ is strictly concave with a unique maximizer $q^m \in [q_L, q_H]$ denoted as *the optimal myopic service effort level*, where $\pi'(q^m) = 0$, since it only maximizes the current period profit. Therefore, the long-run discounted profit of the firm with the service effort decision path $\tilde{Q} = (q_0, q_1, \cdots)$ is given as

$$W(\tilde{Q}) = \pi(q_0) + \sum_{t=1}^{\infty} \delta^t \left(\prod_{l=0}^{t-1} ((\kappa+\tau)F(q_l) - \tau + 1) \right) \pi(q_t). \tag{4.5.1}$$

Given a constant service effort policy $q_t = q,\ t \geq 0$, the long-run discounted profit is given as

$$W(q) = \frac{\pi(q)}{1 - \delta\left((\kappa+\tau)F(q) - \tau + 1\right)}$$

which is increasing in κ and δ, while decreasing in τ.

We investigate the optimal service effort policy in the above stylized model. To facilitate the analysis, we assume the distribution function $F(\cdot)$ satisfies the following *generalized decreasing failure rate* property (GDFR), formulated as

$$\frac{\partial}{\partial q}\left(\frac{\delta(\kappa+\tau)F_1(q)}{1 - \delta\left((\kappa+\tau)F(q) - \tau + 1\right)} \right) \leq 0$$

where $F_1(\cdot)$ is the probability density function of $F(q)$. A sufficient condition for the above GDFR property to hold is $F(q)$ with the property of *decreasing failure rate* (DFR), such as Gamma and Weibull distributions. The GDFR property guarantees that there exits a unique maximizer of $W(q)$, denoted as q^c as *the optimal constant* *service effort level*, since the first order derivative of $W(q)$ is

$$W'(q) = \frac{\pi(q)}{1-\delta\left((\kappa+\tau)F(q)-\tau+1\right)}\left(\frac{\pi'(q)}{\pi(q)}+\frac{\delta(\kappa+\tau)F_1(q)}{1-\delta\left((\kappa+\tau)F(q)-\tau+1\right)}\right)$$

where $\frac{\pi'(q)}{\pi(q)}$ is decreasing in q due to concavity of $\pi(q)$ and the second term is decreasing in q based on GDFR property. Obviously, we have $W'(q^m) \geq 0$. Therefore, there exists a unique service effort level q^c, such that $W'(q^c) = 0$, and $q^c \geq q^m$. To reduce the complexity of the analysis, we assume the unique maximizer is in the interior as $q^c \in [q_L, q_H]$.

Based on the assumption of the demand dynamics, the optimal service effort policy exists in the above problem. We investigate whether a constant service effort policy or an oscillated service effort policy is optimal in the above profit maximizing problem. It turns out that the constant service effort policy is optimal. Formally, we have the following result:

Proposition 4.1. *If consumers are myopic, the optimal service effort policy is a constant policy* $q_t = q^c \geq q^m,\ t \geq 0$, *where* q^c *is the unique maximizer of* $W(q)$. *The optimal service effort level* q^c *is increasing in* κ *and* τ, *as well as* δ. *Besides,* $\frac{\pi(q^m)}{1-\delta((\kappa+\tau)F(q^m)-\tau+1)} \leq W(q^c) \leq \frac{\pi(q^m)}{1-\delta(\kappa+1)}$ *and* $W(q^c)$ *is increasing in* κ *and* δ, *while decreasing in* τ.

The above result is straightforward and intuitive. The managerial implication is that, the firm should adopt a constant service effort policy in order to maximize the long-run discounted profit if consumers are myopic. A constant service effort policy smooths the demand in each period; an oscillated service effort causes variability in demand, which reduces the long-run discounted profit. Formally, suppose the firm adopts the following alternative service effort policy $(q_0, q_1, q^c, \cdots)$, where $q_0 \leq q^c \leq q_1$. It can be checked that the long-run discounted profit from the alternative service effort policy is smaller than that from the constant policy with $q_t = q^c$.

Therefore, if consumers are backward-looking and their purchase decisions are influenced by satisfaction from the most recent experiences, the firm should provide a larger service effort compared with the myopic policy. Due to social interactions among consumers, the firm should provide a higher service effort if the positive or the negative social interaction effect is strong. On the one hand, more consumers will be satisfied with higher service experiences, where more demand will be attracted; on the other hand, fewer consumers will be dissatisfied due to higher service experiences, where less potential consumers will be lost. However, a larger negative social interaction intensity always leads to a smaller profit. The discount factor δ measures the levels of importance of the profits in future periods in operational target. A higher δ indicates the manager trades off more short-term profit for the long-term benefit from a large proportion of satisfied consumers.

The market coverage under the above constant service effort policy depends on the level q^c. If $(\kappa+\tau)F(q^c)-\tau>0$, the firm will gradually acquire consumers and eventually cover the whole market; if $(\kappa+\tau)F(q^c)-\tau=0$, the firm maintains the initial market size all the time; while if $(\kappa+\tau)F(q^c)-\tau<0$, the firm will systematically lose consumers and eventually exit the market.

4.6 Service Effort Decision with Adaptive Consumers Under Social Interactions

In this section, we focus on the service effort policy with adaptive consumers, where their purchase decisions are subject to satisfaction depending on past experiences and adaptively formed expectations under the influence of social interactions. Similar to the benchmark model, we also assume the single period profit only depends on the service effort level as $\pi(q)$, which is strictly concave with a unique maximizer $q^m \in [q_L, q_H]$, where $\pi'(q^m)=0$, as the myopic service effort policy. Consumer overall experience is updated based on the exponential smoothing rule. Therefore, the long-run discounted profit optimization problem with initial overall experience r is given as

$$V(r)=\max_{q\in[q_L,q_H]} \pi(q)+\delta\left((\kappa+\tau)F(q,r)-\tau+1\right)V(\gamma r+(1-\gamma)q). \tag{4.6.1}$$

Given the initial overall experience r, if the firm adopts a constant service effort policy $q_t=r,\ t\geq 0$, we have

$$U(r)=\frac{\pi(r)}{1-\delta\left((\kappa+\tau)F(r,r)-\tau+1\right)}$$

which is increasing in κ and δ, while decreasing in τ.

We have the following result in terms of the boundary of the value function:

Lemma 4.1. *The value function $V(r)$ is decreasing in r, increasing in κ while decreasing in τ. Besides, $U(r)\leq V(r)\leq \frac{\pi(q^m)}{1-\delta(\kappa+1)}$.*

Proof. The decreasing property of $V(r)$ is due to the decreasing property of $F(q,r)$ in terms of r. Since $(\kappa+\tau)F(q,r)-\tau+1$ is increasing in κ while decreasing in τ, the value function $V(r)$ is increasing in κ while decreasing in τ. Similarly, $U(r)$ is the long-run discounted profit with the constant service effort policy $q_t=r$

for any $t \geq 0$, and $\frac{\pi(q^m)}{1-\delta(\kappa+1)}$ is the long-run discounted profit where the single-period profit is maximized and all consumers are satisfied in each period. Therefore, $U(r) \leq V(r) \leq \frac{\pi(q^m)}{1-\delta(\kappa+1)}$. $\square$

The result indicates that starting with a higher overall experience, the monopolist will get a lower long-run discounted profit due to the reference effect on consumer expectations which impact their satisfaction. The reason is straightforward, since given service experiences which are determined by service effort decision, the proportion of satisfied consumers will be small if the overall experience is high, since consumer expectations are high. The result also indicates the long-run profit is increasing in the positive social interaction intensity, while decreasing in the negative social interaction intensity. Given the fixed proportion of satisfied consumers, a large positive social interaction intensity will attract more consumers to purchase the service; while a large negative social interaction intensity will reduce the potential demand.

Similar to the previous section, we denote the following term

$$\frac{\delta(\kappa+\tau)f(q)}{1-\delta\left((\kappa+\tau)F(q,q)-\tau+1\right)}$$

as the *generalized failure rate* (GFR), where $f(q) = \frac{\partial F(q,q)}{\partial q} = F_1(q,q) + F_2(q,q)$. We assume the above term is decreasing in q, as the DGFR property. Based on the DGFR property, there exists a unique maximizer q^c as *the optimal constant service effort level* of $U(q)$, such that $q^c \geq q^m$ and q^c is decreasing in κ and τ, while increasing in δ. We assume $q^c \in [q_L, q_H]$. Therefore, q^c satisfies

$$\pi'(q^c) = -\frac{\delta(\kappa+\tau)f(q^c)\pi(q^c)}{1-\delta\left((\kappa+\tau)F(q^c,q^c)-\tau\right)} = -\delta(\kappa+\tau)f(q^c)U(q^c) \leq 0$$

4.6.1 Steady State Service Effort Level

We consider the optimal service effort decision that maximizes the long-run discounted profit as *the strategic policy.* We investigate the optimal service effort decision and its dynamics from the above model. Let $q^\star(r)$ denote the optimal service effort level when the overall experience is r, and $s^\star(r) = \gamma r + (1-\gamma)q^\star(r)$ as the corresponding optimal overall experience. The optimal *service effort decision path* is $q_t^\star = q^\star(r_t^\star)$ and the overall experience dynamic is $r_{t+1}^\star = s^\star(r_t^\star)$, denoted as the *state path.*

Since $(\kappa+\tau)F(q,r)-\tau+1$ is increasing in q, while $V(\gamma r+(1-\gamma)q)$ is decreasing in q since the value function is always decreasing, direct comparisons between $q^\star(r)$ and q^m are not straightforward. We first investigate the characteristics of the steady state service effort level denoted as $q^{\star\star}$ in the strategic policy. In the steady state, the service effort decision will be always $q^{\star\star}$, and the overall experience is also $r = q^{\star\star}$. We have $q^{\star}(q^{\star\star}) = q^{\star\star}$ and $s^{\star}(q^{\star\star}) = q^{\star\star}$ in the steady state. In the following section, we denote $H(q,r) = (\kappa+\tau)F(q,r) - \tau + 1$ for simplicity, with the partial derivative in terms of q and r denoted as $H_1 = (\kappa+\tau)F_1$, $H_2 = (\kappa+\tau)F_2$ and $\delta H < 1$. The condition at q^c is simplified as

$$\frac{\pi'(q)}{U(q)} + \delta\left(H_1(q,q) + H_2(q,q)\right)|_{q=q^c} = 0.$$

We have the following result in terms of the steady state service effort level $q^{\star\star}$ (if exists):

Proposition 4.2. *If a steady state service effort level $q^{\star\star}$ exists, then it is larger than the myopic service effort level and the optimal constant service effort level, i.e., $q^{\star\star} \geq q^c \geq q^m$.*

The above result indicates if the long-run discounted profit optimization problem admits a steady state, the steady state service effort level and the corresponding overall experience will be larger than the optimal constant service effort decision as well as the myopic service effort decision. In Aflaki and Popescu (2012), the steady state service effort level in their model is always less than the optimal constant service effort level. The result in our model is different from theirs, due to the reference effect in consumer expectation formation, which impacts consumer satisfaction thus their repurchase decisions.

Starting from overall experience $r = q^c$, the optimal constant service effort policy $q_t = q^c$ will yield $U(q^c) = \frac{\pi(q^c)}{1-\delta((\kappa+\tau)F(q^c,q^c)-\tau+1)}$ which is smaller than the long-run discounted profit $V(q^c)$ generated from the optimal service effort policy, since the constant service effort policy is only one feasible policy. The above result directly implies the following boundary of $V(r)$ starting with $r = q^{\star\star}$:

Remark 4.1. $U(q^{\star\star}) \leq V(q^{\star\star}) \leq U(q^c) \leq V(q^c)$, since q^c is the optimal constant service effort policy, which indicates $V(q^{\star\star}) \leq U(q^c)$.

Intuitively, the steady state service effort level will be higher if the positive social interaction intensity κ is larger, since the sales quantity will be larger due to a larger positive social interaction effect. However, due to the reference effect on consumer expectations, a higher service effort will induce a higher service experience, which will lead to a higher overall experience thus a higher service expectation (in the stochastic sense), which will decrease the proportion of satisfied consumers. Therefore, the impact of κ on the steady state service effort level depends on the two opposite forces. Similarly, there are two countering forces driving the impact of the negative social interaction effect on the steady state. The following section investigates the impact of various factors on the steady state service effort level. In terms of the impact of social interaction intensities, discount factor and the memory factor on the steady state effort decision, we have the following result:

Corollary 4.1. *The steady state service effort level in the strategic policy $q^{\star\star}$ (if exists) is increasing in κ, τ, γ and δ.*

The above result indicates, due to the social interaction effect (both positive and negative), the steady state service effort level will be larger if social interaction intensities are larger. Therefore, although consumers adaptively form higher service expectations (in the stochastic sense), due to the reference effect, the steady state service effort level should increase if social interactions become more intense. Therefore, both the reference effect in consumer service expectations and the influence of social interactions demand a higher service effort decision in the consumer market.

The monotonicity of the steady state service effort in terms of the discount factor δ indicates managers should balance the tradeoff between the short term profit and the long term benefit, since the single period profit $\pi(q^{\star\star})$ will decrease if δ increases. As discussed before, the overall experience is influenced by all the previous transactions, while the service effort in service delivery determines the most recent service experience. The monotonicity of the steady state service effort in terms of the memory factor γ indicates the firm should exert a high level of service effort if consumers rely more on their previous experiences when updating their overall service experiences. In the sense that, if consumers have a high memory in their past experiences, the monopolist should exert a high level of service effort in service delivery; while if consumers become memoryless, where only the recent service experiences dominate their overall experiences, a slightly lower level of service effort can be exerted. The reason is due to the impact of overall experience in consumer service expectations. A small γ indicates consumer expectations before purchasing the service are less influenced by their previous experiences, while influenced more by their most recent experience. Under the extreme situation where $\gamma \to 0$, the firm can provide the service effort level as low as q^c if consumer expectations are only determined by their most recent experiences; while under $\gamma \to 1$ where in each period, consumers form their expectations only because of their initial experiences r_0, the firm should exert a higher level of service effort in the steady state. The monotonicity of the steady state service effort level is contrary to the result in Aflaki and Popescu (2012), due to the reason that consumer service expectations are reference-dependent.

In the following section, we investigate the existence and uniqueness of the steady state service effort level. We first investigate the monotonicity of the overall experience path $r_t^{\star}$ under the strategic policy. The following result indicates the overall experience path is monotonic:

Lemma 4.2. *Starting with any initial overall experience $r_0 \in [q_L, q_H]$, the optimal overall experience path $r_t^\star, t \geq 0$ monotonically converges to the unique steady state level $q^{\star\star}$.*

The above result indicates, starting with any initial overall experience r_0, the overall experience path will monotonically converge to the unique steady state. Specifically, if the initial experience is large, such that $r_0 \geq q^{\star\star}$, the optimal state path of overall experience $r_t^\star$ will decreasingly converge to $q^{\star\star}$; while if the initial experience is small $r_0 \leq q^{\star\star}$, the optimal state path $r_t^\star$ will increasingly converge to $q^{\star\star}$. Correspondingly, if the initial experience r_0 is larger, the firm should lower the service effort level such that $q_t^\star \leq r_t^\star$ to reduce consumer overall experience systematically, thus increasing the proportion of satisfied consumers to maximize the long-run discounted profit; while if the initial experience r_0 is smaller, the firm should increase the service effort level such that $q_t^\star \geq r_t^\star$ to increase consumer overall experience systematically. The optimal service effort decision with overall experience r_t is $q_t^\star = q^\star(r_t) = \frac{r^\star(r_t)-\gamma r_t}{1-\gamma}$. However, the optimal service effort path may not be monotonic. The following result summarizes the above discussion:

Corollary 4.2. *If the initial experience $r_0 \geq q^{\star\star}$, the optimal service effort level will be less than the overall experience, i.e., $q^\star(r_t) \leq r_t$, and $(1-\gamma)(q_{t+1}^\star - q_t^\star) \leq \gamma(r_t - r_{t+1})$; while if the initial experience $r_0 < q^{\star\star}$, the optimal service effort level will be larger than the overall experience, i.e., $q^\star(r_t) \geq r_t$, and $(1-\gamma)(q_{t+1}^\star - q_t^\star) \geq \gamma(r_t - r_{t+1})$.*

In the following numerical study, we assume the proportion of satisfied consumers under the overall experience r and the service effort q is $\forall q, r \in [0,1],\ F(q,r) = 1-(1+r^2)\exp(-q-a) \in (0,1),\ a \geq 1$, which can be checked with all the required properties of $F(q,r)$, such as $F_1 > 0,\ F_2 < 0,\ F_{22} > 0,\ F_{12} \geq 0,\ F_1(q,q)+F_2(q,q) \geq 0,\ f'(q) \leq 0$. Besides, the GDFR property is also satisfied for the feasible κ and τ. Fig.4.6.1 shows the steady state service effort level with respect to $\kappa \in [0,1]$ and $\tau \in [0,1]$ for fixed $\gamma = 0.5$ and $\delta = 0.4$, and Fig.4.6.2 illustrates the steady state service effort level with respect to $\gamma \in [0,1]$ and $\delta \in [0,0.5]$ for fixed $\kappa = 0.5$ and $\tau = 1$. The single period profit function is assumed as $\pi(q) = -q^2+q+1$, and $F(q,r) = 1-(1+r^2)\exp(-q-2)$.

4.6.2 Market Coverage

The steady state service effort level also impacts the firm market coverage in the long run. Given the initial demand or market size d_0, in period t, the total number

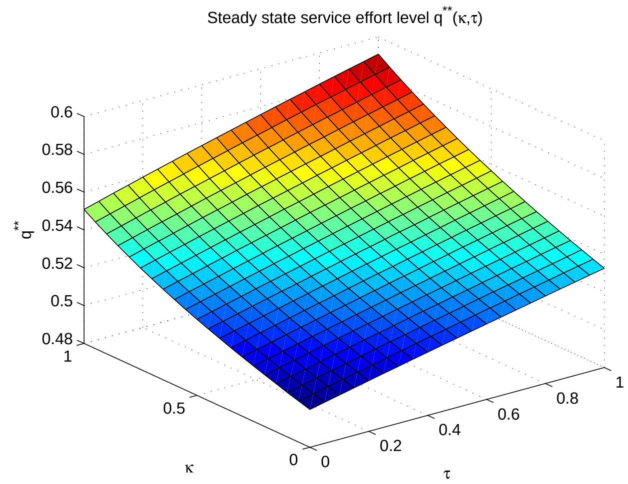

Fig. 4.6.1: Steady state service effort level with respect to the positive social interaction intensity κ and the negative social interaction intensity τ.

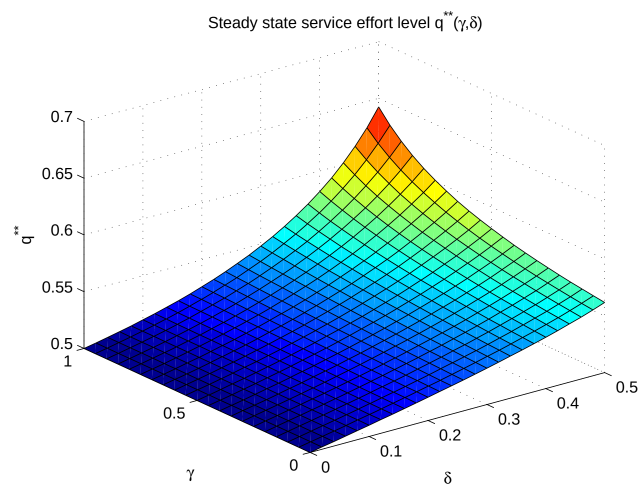

Fig. 4.6.2: Steady state service effort level with respect to the memory factor γ and the discount factor δ.

of served consumers will be

$$d_t = d_0 \prod_{\tau=0}^{t-1} ((\kappa + \tau) F(q_\tau, r_\tau) - \tau + 1)$$

Therefore, depending on the steady state service effort level, we have the following three situations in terms of the market coverage:

- *Market expansion*, i.e., the firm will gradually cover the whole market if $(\kappa + \tau)F(q^{\star\star}, q^{\star\star}) - \tau > 0$.
- *Market contraction*, i.e., the firm will gradually reduce the market size if $(\kappa + \tau)F(q^{\star\star}, q^{\star\star}) - \tau < 0$;
- *Market preservation*, i.e., the firm will keep the market share as large as its initial market coverage in the long run if $(\kappa + \tau)F(q^{\star\star}, q^{\star\star}) - \tau = 0$.

Therefore, we can see the steady state service effort determines the market coverage policy of the firm. Under the market expansion situation, in the long run with an open market, the firm will get infinite demand in the extreme case. In the market with a fixed size, the result should be interpreted as the situation where the firm eventually covers the whole market as a monopolist. Indeed, in reality, we always observe that one dominant supplier covers a large market in a certain product or service category. The result from our model may provide an explanation. Similarly, under the market contraction situation, the firm will gradually lose the market, and eventually exit the market in the long run. The result can also be interpreted as the situation where firms eventually declare bankruptcy and exit the market. In reality, we always observe that some firms eventually disappear from the market due to bankruptcy, mergers or acquisition by others. The market contraction may also explain that it may be the optimal decision for some firms to restrict their market size to focus on a particular market niche. Under the market preservation case, the firm will keep its initial market in the long run. In reality, we can find that some firms only focus on a particular consumer market or conduct business with certain consumer segmentation with similar characteristics. The market preservation may provide an alternative mechanism from the social interaction perspective.

Since the steady state service effort level is increasing in κ, τ, γ and δ, based on our model, these factors impact the firm market coverage policy. Based on the assumption that $F(q, q)$ is monotonic in $q \in [\underline{q}, \bar{q}]$, given κ and τ, there exists a unique threshold service effort level $q^S = q(\kappa, \tau)$ for the market expansion case, such that

$$(\kappa + \tau)F(q^S, q^S) - \tau = 0 \Rightarrow F(q^S, q^S) = \frac{\tau}{\kappa + \tau}$$

where the RHS is decreasing in κ while increasing in τ. Therefore, based on the implicit function theorem, we have q^S is decreasing in κ while increasing in τ. Therefore, compared with the monotonicity of the steady state service effort level, we have the following result in terms of the impact of κ, τ, γ and δ on the market coverage policy:

Proposition 4.3. *Given the other factors fixed, if the positive social interaction intensity κ, the memory factor γ, or the discount factor δ is large enough, the firm will eventually cover the whole market. While the impact of the negative social interaction intensity τ on the market coverage is undetermined.*

Proof. The result is straightforward from the comparison between $q^{\star\star}$ and q^S. □

The above result indicates the firm will eventually cover the whole market under the optimal service effort policy if the positive social interaction intensity κ, the memory factor γ, or the discount factor δ is large enough, since the steady state service effort level will become larger than the threshold level. However, the impact of the negative social interaction intensity τ is inclusive. The reason is immediately obvious from the role of τ in the threshold service effort level. The demand dynamic is always decreasing in the negative social interaction effect for any service effort level, which drives the threshold service effort q^S to be large. Although the steady state service effort level $q^{\star\star}$ is also increasing in τ, it is possible that the threshold can not be achieved for any possible $\tau \geq 0$, which implies the market contraction case.

The region of market coverage in terms of κ and τ is depicted in Fig.4.6.3 with the single period profit function $\pi(q) = -q^2 + q + 1$, and $F(q, r) = 1 - (1 + r^2)\exp(-q - 2)$ under $\gamma = 0.5,\ \delta = 0.4$.

From the figure, we can observe that the impact of positive and negative social interaction intensities on the market coverage may not be monotonic, especially when κ and τ are both large. On the one hand, both κ and τ will lead to a higher steady state service effort decision $q^{\star\star}$; on the other hand, a large κ will lead to a smaller q^S, although q^S is increasing in τ. The two forces lead to a non-monotonic impact of κ and τ on the market coverage, especially when both positive and negative social interaction intensities are high.

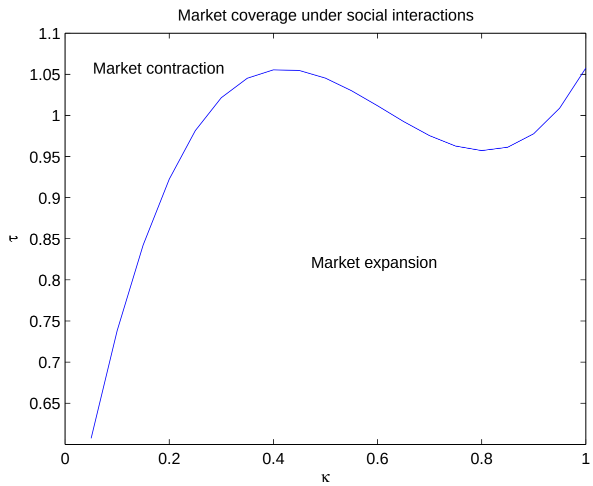

Fig. 4.6.3: Market coverage in terms of the positive social interaction intensity κ and the negative social interaction intensity τ.

4.7 Service Effort Decision with Expectation-dependent Profit Under Social Interactions

4.7.1 Expectation-dependent Profit Function

In previous sections, we assume the single period profit function only depends on the service effort level, where the overall experience only impacts consumer service expectations before purchasing, which further impacts the proportion of satisfied consumers after consuming the service. The assumption is reasonable for the service that each consumer only demands a fixed unit of the service. As discussed in the previous section, service expectations also impact consumer purchase behavior, especially the consumption quantity/volume $k(q, r)$, during service encounters. In this section, we take the impact of consumer expectations on the single period profit into consideration, and investigate the structural property of the optimal service effort policy and its dynamics.

We briefly discuss the impact of expectations on consumer purchase behaviors during service encounters. For an individual consumer who decides to purchase the service, the purchase quantity/volume may depend on service expectation q^E and service experience q^A thus service effort q. Specifically, if service effort thus service experience is smaller than service expectation, the consumer may not purchase the service, or reduce the purchase volume since expectations are not satisfied. While if service effort thus service experience is larger than expectation, the consumer may consume more service during service encounter. In other words, the service expectation q^E serves as a *reference point* for the consumer, and the gap between q^E and q determines the magnitude of reference effect in terms of profit (since the price is exogenous) as $\pi^R(q-q^E,q^E)$. Therefore, the profit from an individual consumer is $\pi(q,q^E)=\pi(q)+\pi^R(q-q^E,q^E)$. Obviously, given q, $\pi(q,q^E)$ is decreasing in q^E (or the gap q^E-q), or $\pi^R(q-q^E,q^E)$ is decreasing in q^E.

The mean of service expectation Q^E as $q^E(r)=\int_{\underline{q}}^{\bar{q}} q^E \mathrm{d}F(q^E,r)$ is increasing in r by the assumption of the FSD property. From firm perspective, the expected profit from each served consumer is

$$\begin{aligned}\Pi(q,r) &= \int_{\underline{q}}^{\bar{q}} \pi(q,q^E)\mathrm{d}F(q^E,r)=\pi(q)+\int_{\underline{q}}^{\bar{q}} \pi^R(q-q^E,q^E)\mathrm{d}F(q^E,r)\\ &= \pi(q)+\pi^R(q-\hat{q}(r),\hat{q}(r)) \qquad (4.7.1)\end{aligned}$$

where $\hat{q}(r)$ is a function of r. Specifically, if $\pi^R(q^E-q,q^E)$ is decreasing in q^E linearly, then $\hat{q}(r)=q^E(r)$; if $\pi^R(q^E-q,q^E)$ is decreasing and convex in q^E, then $\hat{q}(r)\le q^E(r)$ based on Jensen's inequality; if $\pi^R(q-q^E,q^E)$ is decreasing and concave in q^E, then $\hat{q}(r)\ge q^E(r)$. Generally, $\Pi(q,r)$ is decreasing in r.

The above simple analysis investigates the impact of reference effect of consumer expectations on the profit from an individual consumer perspective. In the following section, to reduce the complexity of the analysis, we adopt a simple absolute difference model in the single period profit function to capture the reference effect as

$$\Pi(q,r)=\pi(q)-R(r-q)$$

We assume $R(x)$ is increasing and convex in x, with $R(0)=0$. The convexity captures the increasing marginal effect of service experience on the profit function, i.e., given q, if overall experience r is higher, the profit loss will be more. Therefore, we have the corresponding first and second order derivatives as well as the partial derivative of $\Pi(q,r)$ in terms of q and r as $\Pi_1=\pi'(q)+R'(r-q)$, $\Pi_2=-R'(r-q)\le 0$ and $\Pi_{12}=R''(r-q)\ge 0$, $\Pi_{11}=\pi''(q)-R''(r-q)\le 0$, $\Pi_{22}=-R''(r-q)\le 0$, which indicate the following property of $\Pi(q,r)$:

- $\Pi(q,r)$ is decreasing and concave in r, i.e., for a fixed service effort level, a larger overall experience will reduce the single period profit more;
- $\Pi(q,r)$ is supmodular in (q,r), i.e., the impact of overall experience on the single period profit will increase if overall experience is larger;
- $\Pi(q,q)=\pi(q)$, where $\pi(q)$ is defined the same as that in previous sections, i.e., if overall experience and the service effort level are equal, there is no reference effect in the profit function.

Based on the supermodularity property of $\Pi(q,r)$, we know the optimal single period service effort level $q^M(r)=\arg\max_q \Pi(q,r)$ is increasing in r, which is intuitive and reasonable.

The long-run discounted profit optimization problem is given as

$$V(r) = \max_{q \in [q_L, q_H]} \Pi(q,r) + \delta \left((\kappa+\tau)F(q,r) - \tau + 1 \right) V(\gamma r + (1-\gamma)q) \tag{4.7.2}$$

which is decreasing in r, since $\Pi(q,r)$ is decreasing in r, and $(\kappa+\tau)F(q,r) - \tau + 1$ is decreasing in r.

In the following section, we still denote $H(q,r) = (\kappa+\tau)F(q,r) - \tau + 1$ as in the previous section to simplify the analysis. The long-run discounted profit of the constant service effort policy $q_t = r$, $t \geq 0$ with the initial overall experience r is

$$U(r) = \frac{\Pi(r,r)}{1 - \delta \left((\kappa+\tau)F(r,r) - \tau + 1 \right)} = \frac{\pi(r)}{1 - \delta \left((\kappa+\tau)F(r,r) - \tau + 1 \right)}$$

and based on previous DGFR property, there exists a unique $r = q^c$, such that $U(q^c)$ is the maximum profit under the optimal constant service effort policy. We also assume $q^c \in [q_L, q_H]$, which satisfies $\pi' + \delta U(H_1 + H_2)|_{q=q^c} = 0$.

The firm can adopt a *myopic policy* in service effort decision which only maximizes the current single-period profit, denoted as $q^M(r)$ with the service effort path q^M_t. The steady state service effort level of the myopic policy is denoted as $q^{M\star}$. At steady state, we have $r = q^{M\star}$, $q^M(r) = q^{M\star}$, which satisfies the following condition

$$\pi'(q^{M\star}) + R'(0) = 0$$

indicating $q^{M\star} \geq q^m$, since $R' \geq 0$. Therefore, due to the impact of expectations in consumer purchase volume, the steady state service effort level in the myopic policy is always larger than the myopic service effort level if profit is not influenced by consumer expectations. The long-run discounted profit under the myopic policy is denoted as $V^M(r)$ with the initial overall experience r. The myopic service effort path monotonically converges to the steady state, which can be proved by induction, as shown in the following result:

Lemma 4.3. *The myopic service effort path and the overall experience path converge to the unique steady state service effort level $q^{M\star}$ monotonically, which satisfies $\pi'(q^{M\star}) + R'(0) = 0$.*

Proof. Since $\pi'(q)$ is decreasing if $q \geq q^m$, the steady state (if exists) $q^{M\star}$ is unique. If $q^M_0 = q^M(r_0) \geq r_0$, we have $r_1 \geq r_0$ and $q^M_1 \geq q^M_0$. Suppose $q^M_t \geq q^M_{t-1}$ and $r_t \geq r_{t-1}$. We have $q^M_{t+1} = q^M(r_t) \geq q^M_t = q^M(r_{t-1})$ and $r_{t+1} = \gamma r_t + (1-\gamma)q^M_t \geq \gamma r_{t-1} + (1-\gamma)q^M_{t-1} = r_t$. Therefore, the service effort level path and the overall experience path both increase. Since the decision space is bounded and compact, there exists a unique steady state. Therefore, the service effort level path and the overall experience path will converge to the unique steady state. The other direction can be similarly proved. □

4.7.2 Steady State Service Effort Level

The optimal service effort policy which maximizes the long-run discounted profit is also denoted as the *strategic policy.* We investigate the structural property of the strategic policy in the following section, especially the steady state service effort level and the dynamics of the strategic service effort policy.

Since $\delta U(H_1+H_2) = \pi(q)\frac{\delta(H_1+H_2)}{1-\delta H}$ is decreasing in $q \geq q^m$, we assume $\delta U(H_1+H_2)|_{q=q^c} \geq R'(0)$. Therefore, $q^c \geq q^{M\star}$. The steady state (if exists) of the strategic policy is solved as the previous section, which is larger than the optimal constant service effort decision and the steady state of the myopic policy as shown in the following result:

Proposition 4.4. *If there exists a steady state in the strategic policy, then the steady state service effort level satisfies $q^{\star\star} \geq q^c \geq q^{M\star}$.*

Therefore, similar to the previous section without the impact of expectations on the single period profit function, the steady state in the strategic policy is larger than the optimal constant service effort level and the steady state in the myopic policy.

Compared with the previous section, we have an additional term in the above steady state condition, i.e., $-\frac{(1-\delta H)R'(0)}{1-\gamma\delta H}$ due to the reference effect in the profit function, which serves as additional force for the firm to increase the steady state, i.e., the steady state service effort level $q^{\star\star}$ is larger than that in the previous section. The comparative static analysis in terms of the parameters is given in the following section:

Corollary 4.3. *The steady state service effort level $q^{\star\star}$ is increasing in γ and τ; while the monotonicity in terms of δ and κ is indetermined.*

Proof. It is easy to see that the RHS of steady state condition is decreasing in γ, indicates the steady state is increasing in γ. Since the first term of the RHS is decreasing in τ as in the previous section, and the second term is also decreasing in τ with the derivative as $\frac{(1-\gamma)\delta R'(0)}{(1-\gamma\delta H)^2}(1-F) \leq 0$, this indicates the steady state $q^{\star\star}$ is increasing in τ. The first term of the RHS is decreasing in δ as in the previous section. While the second term $\frac{-(1-\delta H)R'(0)}{1-\gamma\delta H}$ in the RHS is increasing in δ, since the first order derivative in terms of δ is $\frac{(1-\gamma)HR'(0)}{(1-\gamma\delta H)^2} \geq 0$. Therefore, the monotonicity in terms of δ is indetermined. The monotonicity in terms of κ is also indetermined, since the first term in the RHS is decreasing in κ, while the second term is increasing in κ with the derivative as $\frac{(1-\gamma)\delta R'(0)}{(1-\gamma\delta H)^2}F \geq 0$. □

From the above result, we can see the steady state service effort level is also increasing in γ indicating how consumers form their overall experiences thus how consumer expectations are influenced by their experiences influences the steady state service effort level. The steady state increasing in τ indicates a large steady state service effort level should be provided if the negative social interaction effect from dissatisfied consumers is large. However, the monotonicity of the steady state with respect to the discount factor δ and the positive social interaction intensity κ is indetermined. We provide the following reasons. The monotonicity of the steady state $q^{\star\star}$ in terms of the discount factor δ depends on two countering forces: *the discounting effect* and *the reference effect*. The discounting effect refers to the fact that if the discount factor δ becomes larger, i.e., the firm cares more about the future profit, the steady state service effort level $q^{\star\star}$ should become larger. The reference effect refers to the fact that the profit will decrease if the steady state is larger (since $q^{\star\star} \geq q^m$). The monotonicity in terms of κ depends on the relative strength of *the satisfaction effect* and *the reference effect*. The satisfaction effect refers to the fact that since the proportion of satisfied consumers will increase if the steady state increases, the firm should provide a high steady state service effort level. The reference effect refers to the fact that a high service effort level will induce consumers to form even higher overall experiences, thus making it more difficult to serve the consumers and make them satisfied. Therefore, the firm should trade off these counter-forces when making service effort decisions in service delivery.

The existence and the uniqueness of the steady state service effort is guaranteed in the following result:

Proposition 4.5. *The steady state service effort level in the strategic policy is unique which is stated in the above proposition, and the overall experience path monotonically converges to the steady state.*

Proof. The single period profit is supermodular, since $\hat{\Pi}_{12} = -\frac{\gamma}{(1-\gamma)^2}\Pi_{11} + \frac{1}{1-\gamma}\Pi_{12} \geq 0$. We also know that second term in the value function is supermodular. Therefore, the supermodularity of the RHS guarantees that the optimal state $r_{t+1} = r^\star(r_t)$ is increasing in r_t. Therefore, the state path is monotonic by induction. The uniqueness of the steady state is guaranteed by the monotonicity of the steady state condition in the compact decision space. □

Although, the state path monotonically converges to the unique steady state, and the optimal service effort path will also converge to the steady state, similar to the above section, the optimal service effort path in the strategic policy may not be monotonic. It is possible that the firm should vary the service effort levels in order to guarantee a monotonic overall experience path. Therefore, we can see in this section, although consumer purchase quantity or volume is subject to the reference effect, under certain conditions, the structural properties of the dynamic service effort policy may keep the same as in the previous section. The impact of the steady state service effort in the market coverage can be similarly analyzed as in the previous section, which is omitted here.

4.8 Service Effort Decision Under Satisfaction-dependent Social Interactions

4.8.1 Satisfaction-dependent Social Interactions

In previous sections, we assume both the negative and positive social interaction intensities are constant for all consumers, i.e., each satisfied consumer will attract κ potential consumers while each dissatisfied consumer will discourage τ potential consumers to purchase the service. It is assumed that consumer WOM communication activity does not depend on their satisfaction levels. However, in reality, especially in the networked and social economy, a much satisfied consumer may spread his/her experience to others more actively; while a much disappointed consumer may even create web sites or blogs to criticize the seller which definitely has even high influences on other potential consumer purchase decisions. Both satisfied and dissatisfied consumers may even create virtual communities to praise or criticize the sellers online. Therefore, social interaction intensities may depend on consumer satisfaction levels.

In this section, to capture the above phenomenon, we consider a dynamic service effort decision model under satisfaction-dependent social interactions. We first define the satisfaction level in this section which depends on the gap between the actual service experience q^A and consumer service expectation q^E. For an individual consumer, the *satisfaction level* is defined as $SL = \max(q^A - q^E, 0)$ and the *dissatisfaction level* is defined as $DL = \max(0, q^E - q^A) = -SL$. Therefore, if the gap between service experience and expectation is high, the satisfaction level or the dissatisfaction level will be high. A much satisfied consumer will attract more potential consumers to purchase the service; while a much dissatisfied consumer will discourage more potential consumers to purchase the service (Anderson, 1998). Therefore, we assume for an individual consumer with expectation q^E, social interaction intensities are defined as $\kappa^I = \kappa(SL) = \kappa(q^A - q^E)$, $\tau^I = \tau(DL) = \tau(q^E - q^A)$, which are increasing functions of the corresponding satisfaction or dis-satisfaction levels. Since each individual consumer is either satisfied or dissatisfied, we define satisfaction-dependent social interaction intensities as the following based on $q^A = q$,

$$\kappa(q,r) = \int_{\underline{q}}^{q} \kappa(q - q^E)\mathrm{d}F(q^E, r), \ \tau(q,r) = \int_{q}^{\bar{q}} \tau(q^E - q)\mathrm{d}F(q^E, r).$$

Therefore, given the demand size d_t, service experience q_t and overall experience r_t in period t, incorporating the above satisfaction-dependent social interaction intensities, the demand in period $t+1$ is formulated as

$$d_{t+1} = d_t \left[\int_{\underline{q}}^{q_t} \kappa(q_t - q^E)\mathrm{d}F(q^E, r_t) - \int_{q_t}^{\bar{q}} \tau(q_t^E - q_t)\mathrm{d}F(q^E, r_t) + 1 \right] \tag{4.8.1}$$

Given r_t (thus service expectations distribution is fixed), we can see the positive social interaction effect $\kappa(q_t, r_t) = \int_{\underline{q}}^{q_t} \kappa(q_t - q^E)\mathrm{d}F(q^E, r_t)$ will increase in q_t, while the negative social interaction effect $\tau(q_t, r_t) = \int_{q_t}^{\bar{q}} \tau(q_t^E - q_t)\mathrm{d}F(q^E, r_t)$ will decrease in q_t. Both $\kappa(q_t, r_t)$ and $\tau(q_t, r_t)$ are decreasing in r_t. Therefore, a general dynamic service effort decision model with satisfaction-dependent social interaction intensities is formulated as the following dynamic programming model:

$$V(r) = \max_{q \in [q_L, q_H]} \pi(q) + \delta \left((\kappa(q,r) + \tau(q,r))F(q,r) - \tau(q,r) + 1 \right) V(\gamma r + (1-\gamma)q) \tag{4.8.2}$$

which is decreasing in r, since the average social interaction intensity $(\kappa(q,r) + \tau(q,r))F(q,r) - \tau(q,r) + 1$ is increasing in q and decreasing in r. The existence of steady state service effort level and the related structural property can be analyzed similarly to the above sections. We have the following result:

Proposition 4.6. *Suppose $(\kappa(q,r) + \tau(q,r))F(q,r) - \tau(q,r) + 1$ is supermodular in (q,r), concave in q, then there exists a unique steady state service effort level and all overall experience paths monotonically converge to the steady state.*

Proof. Define $((\kappa(q,r) + \tau(q,r))F(q,r) - \tau(q,r)) = H(q,r)$ and reformulate the long-run discounted profit optimization problem as

$$V(r_t) = \hat{\pi}(r_t, r_{t+1}) + \delta \hat{H}(r_t, r_{t+1}) V(r_{t+1})$$

where $\hat{\pi}(r_t, r_{t+1}) = \pi(\frac{r_{t+1} - \gamma r_t}{1-\gamma})$ and $\hat{H}(r_t, r_{t+1}) = H(\frac{r_{t+1} - \gamma r_t}{1-\gamma}, r_t)$. $\hat{\pi}(r_t, r_{t+1})$ is supermodular in (r_t, r_{t+1}) since $\pi(q)$ is concave. The term $\hat{H}(r_t, r_{t+1})V(r_{t+1})$ is supermodular in (q, r), since we have the cross derivative as

$$\frac{1}{1-\gamma}\left(-\frac{\gamma}{1-\gamma}H_{11} + H_{12}\right)V + \left(-\frac{\gamma}{1-\gamma}H_1 + H_2\right)V' \geq 0$$

since $H_{11} \leq 0$, $H_{12} \geq 0$, $H_1 > 0$ and $H_2 < 0$ as well as $V' \leq 0$. Therefore, the overall experience paths monotonically converge to the unique steady state level. □

Therefore, the service effort policy under satisfaction-dependent social interactions may follow the same structure and the optimal service effort policy will lead to a monotonic overall experience path, which will converge to the unique steady state level.

4.8.2 Service Effort Decision Under Satisfaction-dependent Social Interactions

Since satisfaction-dependent social interaction intensities are a general function in terms of the overall experience and the service effort, in the following section, we assume the following special social interaction intensity function as

$$\kappa^K = \kappa(q,r) = \begin{cases} \kappa_l, & q < r \\ \kappa_h, & q \geq r \end{cases}, \ \tau^K = \tau(q,r) = \begin{cases} \tau_h, & q < r \\ \tau_l, & q \geq r \end{cases}$$

where $\kappa_h \geq \kappa_l$ and $\tau_h \geq \tau_l$ since $\kappa(q, r)$ is increasing in q, and $\tau(q, r)$ is decreasing in q. In other words, social interaction intensities depend on the magnitude of overall experience and service experience, i.e., service effort. Compared with overall experience, a higher service experience induces a higher positive social interaction effect and a lower negative social interaction effect; while a lower service experience induces a higher negative social interaction effect and a lower positive social interaction effect.

Therefore, the long-run discounted profit optimization problem is formulated as

$$V^K(r) = \max_{q \in [q_L, q_H]} \pi(q) + \delta H^K(q,r)V^K(\gamma r + (1-\gamma)q) \tag{4.8.3}$$

where the social interaction effect on demand dynamics is

$$H^K(q,r) = \begin{cases} (\kappa_l + \tau_h)F(q,r) - \tau_h + 1, & q < r \\ (\kappa_h + \tau_l)F(q,r) - \tau_l + 1, & q \geq r \end{cases}$$

with $H^h(q,r) = (\kappa_h + \tau_l)F(q,r) - \tau_l + 1 \geq (\kappa_l + \tau_h)F(q,r) - \tau_h + 1 = H^l(q,r)$. The above model is denoted as the dynamic service effort decision model with satisfaction-dependent social interaction effect.

We define the long-run discounted profit with the constant policy $q_t = r, t \geq 0$ as $U^h(r) = \frac{\pi(r)}{1-\delta((\kappa_h+\tau_l)F(r,r)-\tau_l+1)}$ and the optimal long-run discounted profits with (κ_h, τ_l) and (κ_l, τ_h) as $V^h(r)$ and $V^l(r)$ respectively. Since $H^h(q,r) \geq H^l(q,r)$, we always have $V^h(r) \geq V^l(r)$. Therefore, the value function $V^K(r)$ is bounded as

Lemma 4.4. *The value function is bounded as $V^l(r) \leq V^K(r) \leq V^h(r)$.*

Proof. Since $H^h(q,r) \geq H^K(q,r) \geq H^l(q,r)$, the result follows. □

The corresponding steady states service effort level under (κ_h, τ_l) and (κ_l, τ_h) are denoted as $q_h^{\star\star}$ and $q_l^{\star\star}$ respectively. Since $q^{\star\star}$ is increasing in κ and τ based on the previous result in the above section, we have two separate situations: $q_l^{\star\star} \geq q_h^{\star\star}$ and $q_l^{\star\star} < q_h^{\star\star}$. Based on the steady state condition in the previous section, we have the following result in terms of the comparison of the steady state service effort level in the above model:

Lemma 4.5. *If $\kappa_h + \tau_l \geq \kappa_l + \tau_h$, the steady state service effort level satisfies $q_h^{\star\star} \geq q_l^{\star\star}$; while if $\kappa_h + \tau_l < \kappa_l + \tau_h$, the comparison of the steady state service effort level is inconclusive.*

Proof. The result is straightforward from the steady state condition. Specifically, if $\kappa_h + \tau_l \geq \kappa_l + \tau_h$, at (q,q), we have

$$
\begin{aligned}
& \frac{(\kappa_h+\tau_l)(F_1+F_2)}{1-\delta\left((\kappa_h+\tau_l)F-\tau_l\right)} - \frac{(\kappa_h+\tau_l)F_2}{1-\gamma\delta\left((\kappa_h+\tau_l)F-\tau_l\right)} \\
\geq \ & \frac{(\kappa_l+\tau_h)(F_1+F_2)}{1-\delta\left((\kappa_l+\tau_h)F-\tau_h\right)} - \frac{(\kappa_l+\tau_h)F_2}{1-\gamma\delta\left((\kappa_l+\tau_h)F-\tau_h\right)}
\end{aligned}
$$

which indicates $q_h^{\star\star} \geq q_l^{\star\star}$. However, if $\kappa_h + \tau_l \leq \kappa_l + \tau_h$, the above comparison is inconclusive. □

Therefore, the steady state service effort level will be large when the total social interaction intensities are large enough, in the sense that if social interactions become more intense among consumers, the steady state service effort level in the strategic policy will become larger. The initial overall experience r_0 and the steady state service effort level $q_h^{\star\star}$ or $q_l^{\star\star}$ imply the following result:

Proposition 4.7. *If the initial overall experience is small enough, such that $r_0 \leq \min(q_l^{\star\star}, q_h^{\star\star})$ or $q_l^{\star\star} \leq r_0 \leq q_h^{\star\star}$, the optimal overall experience path will converge to $q_h^{\star\star}$ monotonically, and $V^K(r_0) = V^h(r_0)$.*

Proof. If $r_0 \leq \min(q_l^{\star\star}, q_h^{\star\star})$, the optimal state paths are both increasing in $V^l(r_0)$ and $V^h(r_0)$. Therefore, $V^K(r_0)$ will take the optimal path of $V^h(r_0)$ and achieves the same value, i.e., $V^K(r_0) = V^h(r_0)$. The same result holds for the situation if $q_l^{\star\star} \leq r_0 \leq q_h^{\star\star}$. □

The above result indicates if the initial overall experience is small, the firm can always achieve the same result as the intense social interaction case by providing a higher service effort than the overall experience in each period.

However, if the initial overall experience is large, such that $r_0 \geq \max(q_l^{\star\star}, q_h^{\star\star})$ or $q_h^{\star\star} \leq r_0 \leq q_l^{\star\star}$, the optimal value function $V^K(r_0)$ may not achieve $V^h(r_0)$, and the optimal service effort policy is not obvious. It is possible that a constant service effort policy is optimal under certain conditions. We first define the following constraint long-run discounted profit optimization problem with the constraint $q_t \geq r_t, t \geq 0$ as:

$$W^h(r) = \max_{q \geq r} \pi(q) + \delta H^h(q, r) W^h(\gamma r + (1-\gamma)q)$$

Since the optimal state path is monotonic, we make the following assumption:

If the optimal state path is decreasing in the original problem without the constraint, i.e., $r > q^{\star\star}$, we assume the constant service effort policy is the optimal policy in the above constraint problem, i.e., $W^h(r) = U^h(r)$.

Based on the above assumption, we have the following result in terms of the service effort policy under satisfaction-dependent social interactions:

Proposition 4.8. *If $\max(q_l^{\star\star}, q_h^{\star\star}) \leq r$ or $q_h^{\star\star} \leq r \leq q_l^{\star\star}$, the following provides a sufficient condition for the optimality of a constant service effort policy for $V^K(r)$:*

- *if $\max(q_l^{\star\star}, q_h^{\star\star}) \leq r$, for any $q \leq r$,*

$$U^h(r) \geq \pi(q) + \delta\left((\kappa_l + \tau_h)F(q, r) - \tau_h\right) V^h(\gamma r + (1-\gamma)q);$$

- *if $q_h^{\star\star} \leq r \leq q_l^{\star\star}$, for any $q \geq r$,*

$$U^h(r) \geq \pi(q) + \delta\left((\kappa_h + \tau_l)F(q, r) - \tau_l\right) V^l(\gamma r + (1-\gamma)q).$$

From the above result, we conclude that, if the social interaction effect is influenced by consumer satisfaction levels, specifically, the gap between service effort decision and consumer overall service experience, the firm should adopt a constant service effort policy under certain conditions, especially when consumer initial overall experience is high which leads to high service expectations. The underlying forces for the constant service effort policy to be optimal are discussed here. Suppose the firm decides to provide a high service effort to deliver high quality of service to the consumer market. Consumers will adaptively form high service expectations. Although in the short term, the high service effort and high quality of service may attract more consumers due to social interactions, in the long run the firm has to maintain the high service effort level with a high cost thus a low profit margin since consumer expectations have been raised; otherwise, a low service effort decision will lower the service quality which will decrease the demand substantially due to the large negative social interaction effect. It may be optimal for the firm to keep the service effort at the constant level all the time and keep consumer expectation at a consistent level. Therefore, the managerial implication from the above result is that, if the social interaction effect depends on the adaptive consumer satisfaction level which is determined by their experiences and the expectations, the firm should keep the service effort at a stable level to offer a consistent quality of service. Varying service effort may always be suboptimal from a long-run perspective.

4.9 Discussion and Conclusion

For those frequently purchased products or services, expectations and experiences influence consumer satisfaction in service delivery, which substantially affects consumer purchase decisions under social interactions. As both expectations and experiences depend on how services are delivered during service counters through operations, better understanding of the impact of social interactions in consumer purchase decisions is critical for firms to properly manage expectations and experiences through service effort decisions to enhance consumer satisfaction and improve firm performance.

Drawn on extensive economic and marketing literature as well as consumer behavioral theories, we investigate service effort decisions under social interactions, which are captured as the WOM communication among existing and potential consumers. Specifically, existing consumer repurchase decisions are affected by satisfaction which is determined by pre-purchase expectations and post-purchase experiences. Potential consumers are influenced by existing consumers in service expectations and purchase decisions through social interactions. Consumers are adaptive in their service expectations which are influenced by past experiences from frequent interactions with the firm. We investigate under such context, what would be the impact of social interactions on service effort decisions to achieve profit optimization.

In the benchmark model, where consumer expectations are independent of previous experiences, the optimal service effort policy is a constant policy, where the service effort level is kept constant to maximize the total discounted profit. The constant service effort level is always higher than the short term profit maximizing myopic level. If consumers are adaptive in expectations formation or their purchase volumes are affected by satisfaction, the optimal service effort policy induces a monotonic overall experience path which converges to a unique steady state. Due to consumer adaptations in service expectations formation, the steady state service effort level is higher than either the myopic level or the optimal constant level.

Steady state service effort levels are always increasing if social interactions become more intense, especially for the negative social interaction effect. If consumer expectations are more influenced by overall experiences from previous interactions, a higher level of service effort should be exerted. Consumer satisfaction may also affect social interactions, where highly satisfied or dissatisfied consumers may become more active in spreading service experiences. Under certain conditions, the optimal service effort policy follows the same pattern as previous models, such as a monotonic overall experience path and a unique service effort level. It is possible that due to satisfaction-dependent social interactions, it may be optimal for firms to adopt a constant service effort policy.

Our results show that managers who ignore social interactions in consumer purchase decisions, tend to invest less or exert insufficient effort in service delivery, thereby systematically lose revenue due to lower consumer satisfaction from lower level of service quality. Through sacrificing the current profit to offer a high level of service quality by exerting more effort, a large proportion of existing consumers will be satisfied which will lead to a larger demand through social interactions. However, the optimal service effort policy indicates always exceeding consumer expectations through higher service effort may not be optimal if consumers are adaptive in service expectations, that is in order to maximize the long-run profit, firms may need to systematically lower (increase) consumer expectations if their initial expectations are too high (too low). As discussed in the introduction, managers have already adopted such strategies to manage consumer expectations. As stated in Rust and Oliver (2000), "*delighting the consumer 'raises the bar' of consumer expectations, making it more difficult to satisfy the consumer in the next purchasing cycle and hurting the firm in the long run*". The result in our model confirms this argument.

Our result also indicates if social interactions depend on consumer satisfaction, a constant service effort policy may be optimal in the long-run, especially when consumer initial expectations are high. Under such situations, exerting a lower service effort to lower consumer expectations or providing a high level of service quality to meet consumer expectations are always suboptimal from profit optimization perspective. On the one hand, since consumer expectations are already too high, offering an even higher level of service quality through more effort to satisfy consumer expectations increases operational cost, although a larger positive social interaction effect may be expected. On the other hand, providing a lower service effort to lower consumer expectations may reduce satisfaction which will affect future demand through social interactions due to increased proportion of dissatisfied consumers. Therefore, to strike the balance between service quality and quality of experience under social interactions, a constant service effort policy may be desired to offer a consistent quality of service to consumers.

5. CONCLUSIONS

Social interactions become increasingly influential in consumer purchase decisions. Better understanding of the impact of social interactions in operations management will help carry out the decision-making process more effectively to improve profitability and enhance consumer satisfaction.

Focusing on the three core operational decisions of capacity, price, and quality, we investigate how social interactions would impact operational decisions and firm performance in terms of profit, market share, and consumer satisfaction. Based on three research topics, key results and managerial insights are summarized below:

1. In terms of the impact of social interactions on the capacity and price decisions in a competitive market

    - Social interactions can always benefit a monopolist firm, where a large market can be covered with a smaller capacity and a higher price.
    - Whether social interactions can benefit firms in competition depends on both social interaction intensity and competition intensity captured by market size.
    - Ignoring social interactions will lead to potential market and revenue loss due to suboptimal price and capacity decisions in a monopoly market; while in the competitive market, managers need to be aware of social interactions and their interplay with competition in operational decision-making.

2. In terms of the impact of social interactions on the capacity and price decisions for frequently purchased products or services

    - Both strategic and myopic policies in terms of price and capacity lead to monotonic customer base dynamics which converge to a unique steady state under social interactions.
    - More intense social interactions drive firms to charge lower prices, build higher capacities or obtain lower profit margins which lead to larger steady state customer bases.

- Understanding the influence of social interactions, operational decisions and performance can be better improved; under social interactions, operational decisions should be adapted to balance the tradeoff between short-term profit and long-term benefit to achieve profitability and market expansion.

3. In terms of the impact of social interactions on service effort decisions

    - Service effort decisions should be adapted to systematically increase (decrease) consumer overall experiences to properly manage expectations and satisfaction, if initial overall experience is low (high).
    - More intense social interactions lead to more resource investment in operations to deliver high quality service to enhance consumer satisfaction; satisfaction-dependent social interactions may drive firms to offer a consistent level of service quality through a constant level of service effort decisions.
    - Being aware of social interactions and consumer adaptations in service expectations as well as their interplay will help better allocate resources to manage consumer expectations, experiences and satisfaction in service delivery to increase profitability.

To sum up, in viewing radical changes of consumer behaviors and consumption habits due to the profound impact of information technology, through the three research topics, this dissertation has studied the impact of social interactions on operational decisions and performance in terms of capacity, price and quality in a systematical and synthesized manner. This dissertation has contributed to both management practices and academic research. For practical significance, this dissertation has offered fresh insight about the impact of social interactions in operational decisions and firm performance. Based on analytical results, better understanding of the role of social interactions has been derived, which will help managers improve operational decision-making to achieve profitability and market expansion in the new economy. For academic significance, this dissertation has contributed to the discipline of operations management. In addition to the contributions that have already been highlighted in each topic, this dissertation brings out important perspectives for future operations management research. The three topics highlight the imperative of incorporating consumer behaviors into operational decision-making. Effective operational decisions require hearing the voice of consumers, so that product and service delivery can be carried out efficiently. It is crucial to take the changes and the trends of consumer behaviors into operational decision-making before trying to improve service systems. This dissertation also has contributed to the study of social interactions from an operational perspective. The three topics have considered both positive and negative externalities from social interactions. We not only investigate the impact of social interactions on the dynamics of a service system as existing studies, but also address how decision-making can be better improved to utilize social interactions in operations management.

Limitations of this dissertation are obvious. First, like most theoretical studies, we rely on analytical models to investigate the impact of social interactions on operational decisions and firm performance. Although these models are developed based on empirical and experimental evidence, this dissertation suffers from the drawbacks of typical modeling work, such as the assumptions made in the model construction. Second, as a widely observed phenomenon in consumer markets, social interactions have been directly incorporated into consumer purchase decisions, and we do not focus on the underlying factors that drive the influence of social interactions. Third, for analytical tractability, we adopt some simple functional forms to model social interactions in consumer purchase decisions, and consumers are homogeneous in preferences and utilities.

Several promising directions are foreseeable for future research. One possible direction is to empirically examine the influence of social interactions in consumer purchase decisions, where the result can be incorporated into the analytical models to better improve operational decisions. Another possible direction is to extend the models in several ways. We can consider service competition among asymmetric firms with different capacity costs, social interaction intensities or service values. Intuitively, more efficient firms with a lower capacity cost, or firms with higher social interaction intensities will cover a larger market. Service competition under social interactions can also be extended to dynamic settings. Extensions on the model of social interactions are also promising. More sophisticated functional forms such as a nonlinear relationship between customer base and consumer utility can be employed in the analysis. For example, a concave-convex function with an inflection point (threshold) can be adopted. Analytical models can also be extended to heterogeneous consumers in terms of utilities and preferences. For example, consumers may be heterogeneous in terms of price and service speed sensitivities. It is possible that under social interactions, firms may adopt different operational strategies to serve different types of consumers. For example, some firms may focus on service speed to be more efficient while charging higher prices; while others may focus on lower prices with less efficient service speed. Consumer satisfaction may be impacted by price, service speed and other factors, which also can be extended for future research to investigate the impact of social interactions. Hopefully, this dissertation will open up new avenues for operations management research in the digital age. We expect more investigations incorporating social interactions and other new trends of consumer behaviors in the operations management area.

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6. APPENDIX

6.1 Appendix I: Proofs in Chapter 2

Proof of Proposition 2.1

Proof. Obviously, the optimal price and service rate decision will reduce consumer expected surplus to be the same as the reservation value 0; otherwise, the firm can increase its profit by charging a higher price or providing a lower service rate while keeping the arrival rate unchanged. Therefore, the monopoly profit optimization problem is reduced as

$$\max_{\lambda \in [0,\Lambda], \lambda < \mu} \left(v + \alpha\lambda - CW(\lambda, \mu) - \beta\mu\right)\lambda \tag{6.1.1}$$

where for fixed $\lambda \in [0, \Lambda]$, the profit function is concave in μ, and the optimal service rate is $\mu(\lambda) = \lambda + \sqrt{\frac{w}{\beta}}$. The problem is reduced as

$$\max_{\lambda \in [0,\Lambda]} \left(v + (\alpha - \beta)\lambda - 2\sqrt{\beta w}\right)\lambda \tag{6.1.2}$$

Therefore, if $\alpha \geq \beta$, the profit function is convex and the optimal arrival rate is $\lambda^m = \Lambda$; while if $\alpha < \beta$, the profit function is concave with the optimal arrival rate given as

$$\lambda^m = \min\left(\frac{v - 2\sqrt{\beta w}}{2(\beta - \alpha)}, \Lambda\right).$$

□

Proof of Lemma 2.2

Proof. If $\mu_i > \Lambda$ and $\alpha_i \geq \frac{w}{(\mu_i - \Lambda)^2}$, the expected surplus from firm i is always increasing in $\lambda_i \in [0, \Lambda]$, which indicates for any possible intersection $\lambda_i \in (0, \Lambda)$, $S(\lambda_i, \mu_i, p_i) = S(\Lambda - \lambda_i, \mu_j, p_j) \leq S(\Lambda, \mu_i, p_i)$. Therefore, if $v + \alpha\Lambda - \frac{w}{\mu_i - \Lambda} - p_i \geq$

$\max(S(\Lambda, \mu_j, p_j), 0)$, the market is fully covered by firm i; if $S(\Lambda, \mu_j, p_j) \geq \max(v + \alpha\Lambda - \frac{w}{\mu_i - \Lambda} - p_i, 0)$, the market is fully covered by firm j. □

Proof of Proposition 2.2 and 2.3 To prove Proposition 2.2 and Proposition 2.3, given the other firm's service rate decision μ_2, the profit optimization problem of the firm is given as

$$\begin{aligned}
\max_{\mu_1 \in [\frac{w}{v-p}, \frac{p}{\beta}]} \quad & (p - \beta\mu_1)\lambda_1 \qquad (6.1.3)\\
s.t. \quad & v + \alpha\lambda_1 - p - CW(\lambda_1, \mu_1) \geq 0 \\
& v + \alpha\lambda_1 - p - CW(\lambda_1, \mu_1) = v + \alpha\lambda_2 - p - CW(\lambda_2, \mu_2) \\
& \lambda_1 + \lambda_2 = \Lambda,\ \lambda_i \in [0, \Lambda],\ \lambda_i < \mu_i
\end{aligned}$$

and the Lagrangian function is defined as

$$\begin{aligned}
L(\mu_1, \lambda_1, \lambda_2) \quad = \quad & (p - \beta\mu_1)\lambda_1 + \eta_1\left(v + \alpha\lambda_1 - p - \frac{w}{\mu_1 - \lambda_1}\right) \\
& + \eta_2\left(\alpha\lambda_1 - \frac{w}{\mu_1 - \lambda_1} - \alpha\lambda_2 + \frac{w}{\mu_2 - \lambda_2}\right) + \eta_3(\lambda_1 + \lambda_2 - \Lambda)
\end{aligned}$$

where $\eta_1 \geq 0$, η_2 and η_3 are the Lagrangian multipliers. The necessary condition for the optimal solution is characterized by the following KKT conditions

$$\begin{aligned}
-\beta\lambda_1 + (\eta_1 + \eta_2)\frac{w}{(\mu_1 - \lambda_1)^2} &= 0 \\
p - \beta\mu_1 + \alpha(\eta_1 + \eta_2) - (\eta_1 + \eta_2)\frac{w}{(\mu_1 - \lambda_1)^2} + \eta_3 &= 0 \\
-\alpha\eta_2 + \eta_2\frac{w}{(\mu_2 - \lambda_2)^2} + \eta_3 &= 0 \\
\eta_1\left(v + \alpha\lambda_1 - p - \frac{w}{\mu_1 - \lambda_1}\right) &= 0 \\
\lambda_1 + \lambda_2 - \Lambda &= 0
\end{aligned}$$

from which we can solve the multiplier as

$$\eta_1 = \frac{\beta\lambda_1(\mu_1 - \lambda_1)^2}{w} - \left(\frac{w}{(\mu_2 - \lambda_2)^2} - \alpha\right)^{-1}\left(p - \beta\mu_1 + \alpha\frac{\beta\lambda_1(\mu_1 - \lambda_1)^2}{w} - \beta\lambda_1\right)$$

which is denoted as $\eta_1 = G(\alpha, \lambda_1, \lambda_2, \mu_1, \mu_2)$.

We consider the following two cases $\eta_1 = 0$ and $\eta_1 > 0$ separately. If $\eta_1 > 0$, then $v + \alpha\lambda_1 - p - \frac{w}{\mu_1 - \lambda_1} = 0$. The profit optimization problem is reduced as

$$\begin{aligned}
\max_{\mu_1 \in [\frac{w}{v-p}, \frac{p}{\beta}]} \quad & (p - \beta\mu_1)\lambda_1 \\
s.t. \quad & v + \alpha\lambda_1 - p - \frac{w}{\mu_1 - \lambda_1} = 0 \\
& \alpha\lambda_1 - \frac{w}{\mu_1 - \lambda_1} = \alpha\lambda_2 - \frac{w}{\mu_2 - \lambda_2} \\
& \eta_1 = G(\alpha, \lambda_1, \lambda_2, \mu_1, \mu_2) > 0 \\
& \lambda_1 + \lambda_2 = \Lambda,\ \lambda_i \in [0, \Lambda],\ \lambda_i < \mu_i
\end{aligned} \tag{6.1.4}$$

which is further reduced as

$$\begin{aligned}
\max_{\mu_1 \in [\frac{w}{v-p}, \frac{p}{\beta}]} \quad & \left(p - \beta\left(\lambda_1 + \frac{w}{v - p + \alpha\lambda_1}\right)\right)\lambda_1 \\
s.t. \quad & \alpha\lambda_1 - \frac{w}{\mu_1 - \lambda_1} = \alpha\lambda_2 - \frac{w}{\mu_2 - \lambda_2} \\
& \eta_1 = G(\alpha, \lambda_1, \lambda_2, \mu_1, \mu_2) > 0 \\
& \lambda_1 + \lambda_2 = \Lambda,\ \lambda_i \in [0, \Lambda]\ \lambda_i < \mu_i
\end{aligned} \tag{6.1.5}$$

If $\eta_1 = 0$, then $v + \alpha\lambda_1 - p - \frac{w}{\mu_1 - \lambda_1} \geq 0$, i.e., consumer expected surplus is non-negative. The profit optimization problem is reduced as

$$\begin{aligned}
\max_{\mu_1 \in [\frac{w}{v-p}, \frac{p}{\beta}]} \quad & (p - \beta\mu_1)\lambda_1 \\
s.t. \quad & v + \alpha\lambda_1 - p - \frac{w}{\mu_1 - \lambda_1} \geq 0 \\
& \alpha\lambda_1 - \frac{w}{\mu_1 - \lambda_1} = \alpha\lambda_2 - \frac{w}{\mu_2 - \lambda_2} \\
& \eta_1 = G(\alpha, \lambda_1, \lambda_2, \mu_1, \mu_2) = 0 \\
& \lambda_1 + \lambda_2 = \Lambda,\ \lambda_i \in [0, \Lambda], \lambda_i < \mu_i
\end{aligned} \tag{6.1.6}$$

Without the constraints, based on the monopoly result, the optimal arrival rate is given as $\lambda^m = \min(\lambda^\star, \Lambda)$ and the optimal service rate is given as $\mu^m = \lambda^m + \frac{w}{v - p + \alpha\lambda^m}$. The other firm's profit optimization problem is the same due to symmetry. Suppose the symmetric equilibrium exists. Since each firm adopts the same service rate μ^e and covers $\frac{\Lambda}{2}$, we have Proposition 2.2 if $\eta_1 > 0$ and Proposition 2.3 if $\eta_1 = 0$.

Proof of Proposition 2.2

Proof. The necessary condition is derived from the inequality constraint of $\eta_1 =$

$G(\alpha, \frac{\Lambda}{2}, \frac{\Lambda}{2}, \mu^e, \mu^e) > 0$ in the symmetric equilibrium. We focus on the uniqueness of the symmetric equilibrium. Denote $\Delta_\mu = \frac{w}{v-p+\alpha\frac{\Lambda}{2}}$. Suppose $\mu_2 = \mu^e$, and $\mu_1 = \mu^e + \delta$, where $\delta \geq 0$. In equilibrium, $\lambda_1 = \frac{\Lambda}{2} + \epsilon(\delta)$ and $\lambda_2 = \frac{\Lambda}{2} - \epsilon(\delta)$, where $\epsilon(\delta) \geq 0$. From $\alpha\lambda_1 - \frac{w}{\mu_1-\lambda_1} = \alpha\lambda_2 - \frac{w}{\mu_2-\lambda_2}$, $\epsilon(\delta)$ can be solved from the equation $F(\epsilon,\delta) = 2\alpha\epsilon(\Delta_\mu+\epsilon)(\Delta_\mu+\delta-\epsilon) - 2w\epsilon + w\delta = 0$, where if $\delta = 0$, then $\epsilon = 0$. Based on the implicit function theorem, we have the first order derivative of $\epsilon(\delta)$ with respect to δ as

$$\begin{aligned} \epsilon'(\delta) = -\frac{F_\delta}{F_\epsilon} \\ = \quad -\frac{2\alpha\epsilon(\Delta_\mu+\epsilon)+w}{2\alpha(\Delta_\mu+\epsilon)(\Delta_\mu+\delta-\epsilon)+2\alpha\epsilon(\Delta_\mu+\delta-\epsilon)-2\alpha\epsilon(\Delta_\mu+\epsilon)-2w} \end{aligned}$$

and $\epsilon'(\delta = 0) = -\frac{w}{2\alpha\Delta_\mu^2-2w} = \frac{(v-p+\alpha\frac{\Lambda}{2})^2}{2[(v-p+\alpha\frac{\Lambda}{2})^2-\alpha w]} > 0$. The second order derivative is denoted as $\epsilon''(\delta)$. In the symmetric equilibrium, given the other firm chooses $\mu_2 = \mu^e$, the profit with the service rate decision $\mu_1 = \mu^e+\delta$ is $\pi(\mu^e+\delta, \mu^e) = (p-\beta\mu^e-\beta\delta)(\frac{\Lambda}{2}+\epsilon(\delta))$ and we have

$$\Delta(\delta) = \pi(\mu^e+\delta,\mu^e) - \pi(\mu^e,\mu^e) = (p-\beta\mu^e)\epsilon(\delta) - \frac{\beta\Lambda}{2}\delta - \beta\delta\epsilon(\delta)$$

where the first and second order derivatives with respect to δ are

$$\begin{aligned} \Delta'(\delta) &= (p-\beta\mu^e-\beta\delta)\epsilon'(\delta) - \frac{\beta\Lambda}{2} - \beta\epsilon(\delta) \\ \Delta''(\delta) &= (p-\beta\mu^e-\beta\delta)\epsilon''(\delta) - 2\beta\epsilon'(\delta) \end{aligned}$$

Therefore, if $\Delta'(0) \leq 0$ and $\Delta''(\delta) \leq 0$, the profit function is concave in $\delta \in [0, \frac{p}{\beta} - \mu^e]$, indicating μ^e is the optimal service rate given the other firm chooses $\mu_2 = \mu^e$. □

Proof of Proposition 2.3

Proof. The symmetric equilibrium service rate is solved from $\eta_1 = 0$, where $\lambda_1 = \lambda_2 = \frac{\Lambda}{2}$. We focus on the uniqueness of the equilibrium service rate. Given the other firm chooses $\mu_2 = \mu^e$, if one chooses $\mu_1 = \mu^e+\delta$, the effective arrival rate will be $\lambda = \frac{\Lambda}{2}+\epsilon$. From the equality constraint $\lambda_1+\lambda_2 = \Lambda$ and $\alpha\lambda_1 - \frac{w}{\mu_1-\lambda_1} = \alpha\lambda_2 - \frac{w}{\mu_2-\lambda_2}$, we have

$$\alpha(\frac{\Lambda}{2}+\epsilon)-\frac{w}{\mu^e+\delta-\frac{\Lambda}{2}-\epsilon}=\alpha(\frac{\Lambda}{2}-\epsilon)-\frac{w}{\mu^e-\frac{\Lambda}{2}+\epsilon}$$

where we have $F(\delta,\epsilon)=2\alpha\epsilon\left(\mu^e+\delta-\frac{\Lambda}{2}-\epsilon\right)\left(\mu^e-\frac{\Lambda}{2}+\epsilon\right)-2w\epsilon+w\delta=0$ and if $\delta=0$, then $\epsilon=0$. Based on the implicit function theorem, we have the first order derivative of ϵ with respect to δ as

$$\epsilon'(\delta)=-\frac{2\alpha\epsilon\left(\mu^e-\frac{\Lambda}{2}+\epsilon\right)+w}{2\alpha\left(\mu^e-\frac{\Lambda}{2}+\epsilon\right)\left(\mu^e+\delta-\frac{\Lambda}{2}-\epsilon\right)+2\alpha\epsilon\left(\delta-2\epsilon\right)-2w}$$

and $\epsilon'(\delta=0)=-\frac{w}{2\alpha\left(\mu^e-\frac{\Lambda}{2}\right)\left(\mu^e-\frac{\Lambda}{2}\right)-2w}$. The second order derivative is denoted as $\epsilon''(\delta)$. In the symmetric equilibrium, given the other firm chooses $\mu_2=\mu^e$, the profit with the service rate decision $\mu_1=\mu^e+\delta$ is $\pi(\mu^e+\delta,\mu^e)=(p-\beta\mu^e-\beta\delta)(\frac{\Lambda}{2}+\epsilon)$ and we have

$$\Delta(\delta)=\pi(\mu^e+\delta,\mu^e)-\pi(\mu^e,\mu^e)=(p-\beta\mu^e)\epsilon-\frac{\beta\Lambda}{2}\delta-\beta\delta\epsilon$$

where the first and second order derivatives with respect to δ are

$$\begin{aligned}\Delta'(\delta) &= (p-\beta\mu^e-\beta\delta)\epsilon'(\delta)-\frac{\beta\Lambda}{2}-\beta\epsilon\\ \Delta''(\delta) &= (p-\beta\mu^e-\beta\delta)\epsilon''(\delta)-2\beta\epsilon'(\delta)\end{aligned}$$

It is clear that $\Delta'(0)=0$ which indicates $\mu_1=\mu^e$ is a necessary condition for the equilibrium. Therefore, if $\Delta''(\delta)\leq 0$, the profit function is concave in $\delta\in[0,\frac{p}{\beta}-\mu^e]$, indicating μ^e is the optimal service rate given the other firm chooses $\mu_2=\mu^e$. □

Proof of Proposition 2.4 and 2.5 To prove Proposition 2.4 and 2.5, given the other firm's price decision p_2, the profit optimization problem of the firm is given as

$$\begin{aligned}\max_{p_1}\quad & (p_1-\beta\mu)\lambda_1 && (6.1.7)\\ s.t.\quad & v+\alpha\lambda_1-p_1-CW(\lambda_1,\mu)\geq 0\\ & v+\alpha\lambda_1-p_1-CW(\lambda_1,\mu)=v+\alpha\lambda_2-p_2-CW(\lambda_2,\mu)\\ & \lambda_1+\lambda_2=\Lambda,\ \lambda_i\in[0,\Lambda]\end{aligned}$$

and the Lagrangian function is defined as

$$\begin{aligned} L(p_1, \lambda_1, \lambda_2) &= (p_1 - \beta\mu)\lambda_1 + \eta_1 \left(v + \alpha\lambda_1 - p_1 - \frac{w}{\mu - \lambda_1} \right) \\ &\quad + \eta_2 \left(\alpha\lambda_1 - p_1 - \frac{w}{\mu - \lambda_1} - \alpha\lambda_2 + p_2 + \frac{w}{\mu - \lambda_2} \right) + \eta_3(\lambda_1 + \lambda_2 - \Lambda) \end{aligned}$$

where $\eta_1 \geq 0$, η_2 and η_3 are Lagrangian multipliers. The necessary condition for the optimal solution is characterized by the following KKT condition

$$\begin{aligned} \lambda_1 - (\eta_1 + \eta_2) &= 0 \\ p_1 - \beta\mu + \alpha(\eta_1 + \eta_2) - (\eta_1 + \eta_2)\frac{w}{(\mu - \lambda_1)^2} + \eta_3 &= 0 \\ -\alpha\eta_2 + \eta_2 \frac{w}{(\mu - \lambda_2)^2} + \eta_3 &= 0 \\ \eta_1 \left(v + \alpha\lambda_1 - p_1 - \frac{w}{\mu - \lambda_1} \right) &= 0 \\ \lambda_1 + \lambda_2 - \Lambda &= 0 \end{aligned}$$

from which we can solve the multiplier as

$$\eta_1 = \lambda_1 - \left(\frac{w}{(\mu - \lambda_2)^2} - \alpha \right)^{-1} \left(p_1 - \beta\mu + \alpha\lambda_1 - \frac{\lambda_1 w}{(\mu - \lambda_1)^2} \right) \geq 0$$

which is denoted as $\eta_1 = G(\alpha, \lambda_1, \lambda_2, p_1)$. We consider two cases $\eta_1 > 0$ and $\eta_1 = 0$ separately.

If $\eta_1 > 0$, then $v + \alpha\lambda_1 - p_1 - \frac{w}{\mu - \lambda_1} = 0$, consumer expected surplus is the same as the reservation value. The profit optimization problem is reduced as

$$\begin{aligned} \max_{p_1} \quad & (p_1 - \beta\mu)\lambda_1 \\ s.t. \quad & v + \alpha\lambda_1 - p_1 - \frac{w}{\mu - \lambda_1} = 0 \\ & \alpha\lambda_1 - p_1 - \frac{w}{\mu - \lambda_1} = \alpha\lambda_2 - p_2 - \frac{w}{\mu - \lambda_2} \\ & \eta_1 = G(\alpha, \lambda_1, \lambda_2, p_1) > 0 \\ & \lambda_1 + \lambda_2 = \Lambda, \ \lambda_i \in [0, \Lambda], \ \lambda_i < \mu_i \end{aligned} \tag{6.1.8}$$

which is further reduced as

$$\begin{aligned}
\max_{\mu_1} \quad & \left(v + \alpha\lambda_1 - \frac{w}{\mu - \lambda_1} - \beta\mu\right)\lambda_1 \\
s.t. \quad & \alpha\lambda_1 - p_1 - \frac{w}{\mu - \lambda_1} = \alpha\lambda_2 - p_2 - \frac{w}{\mu - \lambda_2} \\
& \eta_1 = G(\alpha, \lambda_1, \lambda_2, p_1) > 0 \\
& \lambda_1 + \lambda_2 = \Lambda,\ \lambda_i \in [0, \Lambda],\ \lambda_i < \mu
\end{aligned} \tag{6.1.9}$$

where without the constraint, the objective function is concave and the optimal arrival rate satisfies the first order condition

$$v + 2\alpha\lambda_1 - \beta\mu - \frac{w\mu}{(\mu - \lambda_1)^2} = 0$$

Since the profit optimization problems of the two firms are identical, in the symmetric equilibrium, each firm covers $\frac{\Lambda}{2}$. Therefore, the necessary condition for the optimality of $\lambda_1 = \frac{\Lambda}{2}$ is that

$$v + \alpha\Lambda - \beta\mu - \frac{w\mu}{(\mu - \frac{\Lambda}{2})^2} \geq 0,\ \eta_1(\alpha, \frac{\Lambda}{2}, \frac{\Lambda}{2}, p) > 0$$

where the first condition holds since $\Lambda < \Lambda^c$, and from the condition $\eta_1(\alpha, \frac{\Lambda}{2}, \frac{\Lambda}{2}, p) > 0$, we have Proposition 2.4.

If $\eta_1 = 0$, then $v + \alpha\lambda_1 - p_1 - \frac{w}{\mu - \lambda_1} \geq 0$, i.e., consumer expected surplus is non-negative. The profit optimization problem is reduced as

$$\begin{aligned}
\max_{p_1} \quad & (p_1 - \beta\mu)\lambda_1 \\
s.t. \quad & v + \alpha\lambda_1 - p_1 - \frac{w}{\mu - \lambda_1} \geq 0 \\
& \alpha\lambda_1 - p_1 - \frac{w}{\mu - \lambda_1} = \alpha\lambda_2 - p_2 - \frac{w}{\mu - \lambda_2} \\
& \eta_1 = G(\alpha, \lambda_1, \lambda_2, p_1) = 0 \\
& \lambda_1 + \lambda_2 = \Lambda,\ \lambda_i \in [0, \Lambda],\ \lambda_i < \mu
\end{aligned} \tag{6.1.10}$$

where from the equality constraint $\eta_1 = 0$, in the symmetric equilibrium $\lambda_1 = \lambda_2 = \frac{\lambda}{2}$, we have the symmetric equilibrium price $p^e = \beta\mu + \frac{4w\Lambda}{(2\mu - \Lambda)^2} - \alpha\Lambda$. Therefore, if customer expected surplus is non-negative, such that

$$v + \frac{3}{2}\alpha\Lambda - \beta\mu - \frac{w(\mu + \frac{\Lambda}{2})}{(\mu - \frac{\Lambda}{2})^2} \geq 0$$

the above symmetric equilibrium price p^e holds, which leads to Proposition 2.5.

Proof of Proposition 2.4

Proof. Suppose $p^e = v + \alpha\frac{\Lambda}{2} - \frac{w}{\mu - \frac{\Lambda}{2}}$ is a symmetric equilibrium. Then from the condition $\eta_1(\alpha, \frac{\Lambda}{2}, \frac{\Lambda}{2}, p) > 0$, we can solve the condition

$$v + \frac{3}{2}\alpha\Lambda - \beta\mu - \frac{w(\mu + \frac{\Lambda}{2})}{(\mu - \frac{\Lambda}{2})^2} < 0$$

which is decreasing in Λ, since the derivative is $\frac{3}{2}\left(\alpha - \frac{w(\mu+\frac{\Lambda}{2})}{(\mu-\frac{\Lambda}{2})^3}\right) < \frac{3}{2}\left(\alpha - \frac{w}{(\mu-\frac{\Lambda}{2})^2}\right) <$ 0. Therefore, if $\Lambda^{c1} < \Lambda < \Lambda^c$, the above condition always holds. We focus on the uniqueness of the equilibrium price p^e. Given the other firm chooses $p_2 = p^e$, if the firm chooses $p_1 = p^e - \delta$, the effective arrival rate will be $\lambda = \frac{\Lambda}{2} + \epsilon$. From the equality constraint $\lambda_1 + \lambda_2 = \Lambda$ and $\alpha\lambda_1 - p_1 - \frac{w}{\mu - \lambda_1} = \alpha\lambda_2 - p_2 - \frac{w}{\mu-\lambda_2}$, we have

$$\alpha(\frac{\Lambda}{2} + \epsilon) - (p^e - \delta) - \frac{w}{\mu - \frac{\Lambda}{2} - \epsilon} = \alpha(\frac{\Lambda}{2} - \epsilon) - p^e - \frac{w}{\mu - \frac{\Lambda}{2} + \epsilon}$$

where we have $F(\delta, \epsilon) = (2\alpha\epsilon + \delta)\left((\mu - \frac{\Lambda}{2})^2 - \epsilon^2\right) - 2w\epsilon = 0$ and if $\delta = 0$, then $\epsilon = 0$. Based on the implicit function theorem, we have the first order derivative of ϵ with respect to δ as

$$\epsilon'(\delta) = -\frac{F_\delta}{F_\epsilon} = -\frac{\left((\mu - \frac{\Lambda}{2})^2 - \epsilon^2\right)}{2\alpha\left((\mu - \frac{\Lambda}{2})^2 - \epsilon^2\right) - 2(2\alpha\epsilon + \delta)\epsilon - 2w}$$

and $\epsilon'(\delta = 0) = -\frac{(\mu-\frac{\Lambda}{2})^2}{2\alpha(\mu-\frac{\Lambda}{2})^2 - 2w} > 0$. The second order derivative is denoted as $\epsilon''(\delta)$. In the symmetric equilibrium, given the other firm chooses $p_2 = p^e$, the profit with the price $p_1 = p^e - \delta$ is $\pi(p^e - \delta, p^e) = (p^e - \delta - \beta\mu)(\frac{\Lambda}{2} + \epsilon)$ and we have

$$\Delta(\delta) = \pi(p^e - \delta, p^e) - \pi(p^e, p^e) = (p^e - \delta - \beta\mu)\epsilon - \delta(\frac{\Lambda}{2} + \epsilon)$$

where the first and second order derivatives with respect to δ are

$$\begin{aligned}
\Delta'(\delta) &= -2\epsilon + (p^e - 2\delta - \beta\mu)\epsilon'(\delta) - \frac{\Lambda}{2} \\
\Delta''(\delta) &= (p^e - \beta\mu - 2\delta)\epsilon''(\delta) - 4\epsilon'(\delta)
\end{aligned}$$

Therefore, if $(v+\alpha\frac{\Lambda}{2}-\frac{w}{\mu-\frac{\Lambda}{2}}-\beta\mu)\frac{(\mu-\frac{\Lambda}{2})^2}{2w-2\alpha(\mu-\frac{\Lambda}{2})^2}-\frac{\Lambda}{2}\leq 0$, and if $\Delta''(\delta)\leq 0$, the profit function is concave in $\delta\in[0,p^e-\beta\mu]$, which indicates p^e is the optimal price given the other firm chooses $p_2=p^e$, and p^e is the unique Nash equilibrium. □

Proof of Proposition 2.5

Proof. The symmetric equilibrium price p^e is solved from the condition $\eta_1 = 0$ under $\lambda_1 = \lambda_2 = \frac{\Lambda}{2}$. We focus on the uniqueness of the equilibrium price p^e. Given the other firm chooses $p_2 = p^e$, if one chooses $p_1 = p^e - \delta$, the effective arrival rate will be $\lambda = \frac{\Lambda}{2}+\epsilon$. From the equality constraint $\lambda_1+\lambda_2 = \Lambda$ and $\alpha\lambda_1 - p_1 - \frac{w}{\mu-\lambda_1} = \alpha\lambda_2 - p_2 - \frac{w}{\mu-\lambda_2}$, we have

$$\alpha(\frac{\Lambda}{2}+\epsilon)-(p^e-\delta)-\frac{w}{\mu-\frac{\Lambda}{2}-\epsilon}=\alpha(\frac{\Lambda}{2}-\epsilon)-p^e-\frac{w}{\mu-\frac{\Lambda}{2}+\epsilon}$$

where we have $F(\delta,\epsilon) = (2\alpha\epsilon+\delta)\left((\mu-\frac{\Lambda}{2})^2-\epsilon^2\right)-2w\epsilon = 0$ and if $\delta = 0$, then $\epsilon = 0$. Based on the implicit function theorem, we have the first order derivative of ϵ with respect to δ as

$$\epsilon'(\delta)=-\frac{F_\delta}{F_\epsilon}=-\frac{\left((\mu-\frac{\Lambda}{2})^2-\epsilon^2\right)}{2\alpha\left((\mu-\frac{\Lambda}{2})^2-\epsilon^2\right)-2(2\alpha\epsilon+\delta)\epsilon-2w}$$

and $\epsilon'(\delta=0)=-\frac{(\mu-\frac{\Lambda}{2})^2}{2\alpha(\mu-\frac{\Lambda}{2})^2-2w}>0$. The second order derivative is denoted as $\epsilon''(\delta)$. In the symmetric equilibrium, given the other firm chooses $p_2 = p^e$, the profit with the price $p_1 = p^e-\delta$ is $\pi(p^e-\delta,p^e) = (p^e-\delta-\beta\mu)(\frac{\Lambda}{2}+\epsilon)$ and we have

$$\Delta(\delta)=\pi(p^e-\delta,p^e)-\pi(p^e,p^e)=(p^e-\delta-\beta\mu)\epsilon-\delta(\frac{\Lambda}{2}+\epsilon)$$

where the first and second order derivatives with respect to δ are

$$\begin{aligned}\Delta'(\delta) &= -2\epsilon+(p^e-2\delta-\beta\mu)\epsilon'(\delta)-\frac{\Lambda}{2}\\ \Delta''(\delta) &= (p^e-\beta\mu-2\delta)\epsilon''(\delta)-4\epsilon'(\delta)\end{aligned}$$

where if $p^e=\beta\mu+\frac{4w\Lambda}{(2\mu-\Lambda)^2}-\alpha\Lambda$, then $\Delta'(\delta=0)=0$. Therefore, if $\Delta''(\delta)\leq 0$, the profit function is concave in $\delta\in[0,p^e-\beta\mu]$, which indicates p^e is the optimal price given the other firm chooses $p_2=p^e$, and p^e is the unique Nash equilibrium. □

Proof of Lemma 2.7

Proof. If $\alpha \geq \beta$, given the arrival rate $\lambda \in [0, \Lambda]$, the monopolist can set the service rate at $\mu = \lambda + \sqrt{\frac{w}{\beta}}$ to maximize consumer surplus, which is always positive. Therefore, in order to cover the whole market, consumer expected surplus should be non-negative, as

$$S(\Lambda, \mu) = v + \alpha\Lambda - \frac{w}{\mu - \Lambda} - \beta\mu - \Delta \geq 0$$

Since $S(\Lambda, \mu)$ is concave in μ, we can solve the optimal service rate in the range

$$\begin{aligned} \mu^m \quad \in \quad & [\frac{v + (\alpha+\beta)\Lambda - \Delta - \sqrt{(v + (\alpha-\beta)\Lambda - \Delta)^2 - 4\beta w}}{2\beta}, \\ & \frac{v + (\alpha+\beta)\Lambda - \Delta + \sqrt{(v + (\alpha-\beta)\Lambda - \Delta)^2 - 4\beta w}}{2\beta}] \end{aligned}$$

If $\alpha < \beta$, $S^M(\lambda)$ is decreasing in λ. Therefore, if $v + (\alpha-\beta)\Lambda - 2\sqrt{\beta w} - \Delta \geq 0$, the monopolist can still set the corresponding service rate to maximize consumer surplus which is non-negative if the whole market is covered. Therefore, the optimal service rate is still in the above range. While if $v + (\alpha - \beta)\Lambda - 2\sqrt{\beta w} - \Delta < 0$, consumer maximum surplus is negative if the whole market is covered. Therefore, the maximum market can be covered is $\lambda^m = \frac{v - 2\sqrt{\beta w} - \Delta}{\beta - \alpha} < \Lambda$ and the optimal service rate is $\mu^m = \lambda^m + \sqrt{\frac{w}{\beta}}$. □

Proof of Lemma 2.8

Proof. Since $\alpha \geq \beta$, each firm can cover the whole market. Therefore, the market will be fully covered by the two firms. The service rate $\mu = \Lambda + \sqrt{\frac{w}{\beta}}$ will maximize consumer expected surplus if all consumers join the same queue. Given $\mu = \Lambda + \sqrt{\frac{w}{\beta}}$, consumer expected surplus $S(\lambda, \Lambda + \sqrt{\frac{w}{\beta}}) = v + \alpha\lambda - \frac{w}{\Lambda + \sqrt{\frac{w}{\beta}} - \lambda} - \beta(\Lambda + \sqrt{\frac{w}{\beta}}) - \Delta$ is increasing in $\lambda \in [0, \Lambda]$. Therefore, given the other firm chooses $\mu^e = \Lambda + \sqrt{\frac{w}{\beta}}$, if $\mu > \Lambda + \sqrt{\frac{w}{\beta}}$, consumer expected surplus $S(\lambda, \mu)$ is smaller than $S(\Lambda, \Lambda + \sqrt{\frac{w}{\beta}})$ for any $\lambda \in [0, \Lambda]$, since $S(\lambda, \mu) \leq S(\Lambda, \mu) \leq S(\Lambda, \Lambda + \sqrt{\frac{w}{\beta}})$ where the first inequality follows from the increasing property of $S(\lambda, \mu)$ in terms of $\lambda \in [0, \Lambda]$ for fixed $\mu \geq \Lambda + \sqrt{\frac{w}{\beta}}$, and the second inequality follows from the decreasing property of $S(\Lambda, \mu)$ for $\mu \geq \Lambda + \sqrt{\frac{w}{\beta}}$; while if $\mu < \Lambda + \sqrt{\frac{w}{\beta}}$, consumer surplus satisfies $S(\lambda, \mu) \leq S(\lambda, \lambda + \sqrt{\frac{w}{\beta}}) \leq$

$S(\Lambda, \Lambda + \sqrt{\frac{w}{\beta}})$, where the first inequality follows from the increasing property of $S(\lambda, \mu)$ in terms of μ for fixed $\lambda \in [0, \Lambda]$, and the second inequality follows from the increasing property of $S(\lambda, \lambda + \sqrt{\frac{w}{\beta}})$ for $\lambda \in [0, \Lambda]$. Therefore, all the consumers will join the queue with $\mu = \Lambda + \sqrt{\frac{w}{\beta}}$ to maximize their expected surplus. Therefore, given the other firm chooses $\mu^e = \Lambda + \sqrt{\frac{w}{\beta}}$, the best response for the firm is to set $\mu^e = \Lambda + \sqrt{\frac{w}{\beta}}$. The argument also indicates that if the other firm chooses $\mu \neq \Lambda + \sqrt{\frac{w}{\beta}}$, the best strategy is to set $\mu^e = \Lambda + \sqrt{\frac{w}{\beta}}$ and covers the whole market. Therefore, $\mu^e = \Lambda + \sqrt{\frac{w}{\beta}}$ is the unique equilibrium service rate. □

Proof of Proposition 2.6

Proof. For any service rate μ, the maximum consumer surplus is given as $S(\mu - \sqrt{\frac{w}{\alpha}}, \mu) = v + (\alpha - \beta)\mu - 2\sqrt{\alpha w} - \Delta$ which is decreasing in μ. The curve $S(\Lambda - \lambda, \mu)$ depicts consumer surplus from the service provider with service rate μ when the competitor covers λ. Given $\mu^e = \frac{\Lambda}{2} + \sqrt{\frac{w}{\beta}}$, consumer surplus is maximized at $\Lambda - \lambda = \frac{\Lambda}{2} + \sqrt{\frac{w}{\beta}} - \sqrt{\frac{w}{\alpha}} < \frac{\Lambda}{2}$ or $\lambda = \frac{\Lambda}{2} - \sqrt{\frac{w}{\beta}} + \sqrt{\frac{w}{\alpha}} > \frac{\Lambda}{2}$ as $S(\frac{\Lambda}{2} + \sqrt{\frac{w}{\beta}} - \sqrt{\frac{w}{\alpha}}, \frac{\Lambda}{2} + \sqrt{\frac{w}{\beta}})$ since $\alpha < \beta$. $S(\frac{\Lambda}{2} + \sqrt{\frac{w}{\beta}} - \sqrt{\frac{w}{\alpha}}, \frac{\Lambda}{2} + \sqrt{\frac{w}{\beta}})$ is also the maximum point of the curve $S(\Lambda - \lambda, \frac{\Lambda}{2} + \sqrt{\frac{w}{\beta}})$. We also have the maximum surplus at $\lambda = \frac{\Lambda}{2} - \sqrt{\frac{w}{\beta}} + \sqrt{\frac{w}{\alpha}} > \frac{\Lambda}{2}$ as $S(\frac{\Lambda}{2} - \sqrt{\frac{w}{\beta}} + \sqrt{\frac{w}{\alpha}}, \frac{\Lambda}{2} - \sqrt{\frac{w}{\beta}} + \sqrt{\frac{w}{\alpha}} + \sqrt{\frac{w}{\beta}}) < S(\frac{\Lambda}{2}, \frac{\Lambda}{2} + \sqrt{\frac{w}{\beta}}) < S(\frac{\Lambda}{2} + \sqrt{\frac{w}{\beta}} - \sqrt{\frac{w}{\alpha}}, \frac{\Lambda}{2} + \sqrt{\frac{w}{\beta}})$. Since $S(\Lambda - \lambda, \frac{\Lambda}{2} + \sqrt{\frac{w}{\beta}})$ is increasing while $S(\lambda, \lambda + \sqrt{\frac{w}{\beta}})$ is decreasing in $\lambda \leq \frac{\Lambda}{2} - \sqrt{\frac{w}{\beta}} + \sqrt{\frac{w}{\alpha}}$, $S(\frac{\Lambda}{2}, \frac{\Lambda}{2} + \sqrt{\frac{w}{\beta}})$ is the unique intersection based on the assumption that $S(0, \frac{\Lambda}{2} + \sqrt{\frac{w}{\beta}}) > v + (\alpha - \beta)\Lambda - 2\sqrt{\beta w} - \Delta = S(\Lambda, \Lambda + \sqrt{\frac{w}{\beta}})$. Therefore, if each service provider sets $\mu^e = \frac{\Lambda}{2} + \sqrt{\frac{w}{\beta}}$, each service provider will cover one half of the market and consumers will not deviate with strictly positive surplus.

We next prove the service rate $\mu = \frac{\Lambda}{2} + \sqrt{\frac{w}{\beta}}$ is the best response if the other firm sets $\mu^e = \frac{\Lambda}{2} + \sqrt{\frac{w}{\beta}}$. Given the other service provider sets $\mu^e = \frac{\Lambda}{2} + \sqrt{\frac{w}{\beta}}$, if one sets $\mu' < \frac{\Lambda}{2} + \sqrt{\frac{w}{\beta}}$, we have $\mu' - \sqrt{\frac{w}{\alpha}} < \frac{\Lambda}{2} + \sqrt{\frac{w}{\beta}} - \sqrt{\frac{w}{\alpha}} < \frac{\Lambda}{2}$, $\Lambda - \mu' + \sqrt{\frac{w}{\alpha}} > \frac{\Lambda}{2}$, and $S(\Lambda - \mu' + \sqrt{\frac{w}{\alpha}}, \frac{\Lambda}{2} + \sqrt{\frac{w}{\beta}}) < S(\frac{\Lambda}{2}, \frac{\Lambda}{2} + \sqrt{\frac{w}{\beta}})$ since $S(\Lambda - \lambda, \frac{\Lambda}{2} + \sqrt{\frac{w}{\beta}})$ is increasing in $\lambda \leq \frac{\Lambda}{2} - \sqrt{\frac{w}{\beta}} + \sqrt{\frac{w}{\alpha}}$. Therefore, we have $S(\Lambda - \mu' + \sqrt{\frac{w}{\alpha}}, \frac{\Lambda}{2} + \sqrt{\frac{w}{\beta}}) < S(\frac{\Lambda}{2}, \frac{\Lambda}{2} + \sqrt{\frac{w}{\beta}}) < S(\frac{\Lambda}{2} + \sqrt{\frac{w}{\beta}} - \sqrt{\frac{w}{\alpha}}, \frac{\Lambda}{2} + \sqrt{\frac{w}{\beta}}) < S(\mu' - \sqrt{\frac{w}{\alpha}}, \mu')$ and $S(\frac{\Lambda}{2}, \mu') < S(\frac{\Lambda}{2}, \frac{\Lambda}{2} + \sqrt{\frac{w}{\beta}})$. Therefore, there exists at least one arrival rate $\lambda^e \in (\mu' - \sqrt{\frac{w}{\alpha}}, \frac{\Lambda}{2})$, such that $S(\lambda^e, \mu') = S(\Lambda - \lambda^e, \frac{\Lambda}{2} + \sqrt{\frac{w}{\beta}})$. In the range $\lambda \in [\frac{\Lambda}{2}, \Lambda]$, we always have $S(\lambda, \mu') < S(\lambda, \lambda + \sqrt{\frac{w}{\beta}}) < S(\Lambda - \lambda, \frac{\Lambda}{2} + \sqrt{\frac{w}{\beta}})$, i.e., $S(\lambda, \mu')$ is under the curve $S(\lambda)$, while $S(\lambda)$ is under the curve $S(\Lambda-\lambda, \frac{\Lambda}{2}+\sqrt{\frac{w}{\beta}})$. Therefore, there is no intersection between the curve $S(\lambda,\mu')$ and $S(\lambda, \frac{\Lambda}{2}+\sqrt{\frac{w}{\beta}})$ if $\lambda \in [\frac{\Lambda}{2}, \Lambda]$. Therefore, $\mu' < \frac{\Lambda}{2}+\sqrt{\frac{w}{\beta}}$ is not a best response, since a small service service rate will cover a small market.

Given the other sets $\mu = \frac{\Lambda}{2}+\sqrt{\frac{w}{\beta}}$, if one sets $\mu'' > \frac{\Lambda}{2}+\sqrt{\frac{w}{\beta}}$, we have $\forall\lambda \in [\frac{\Lambda}{2}, \Lambda]$, $S(\lambda,\mu'') \leq S(\lambda, \lambda+\sqrt{\frac{w}{\beta}}) < S(\Lambda-\lambda, \frac{\Lambda}{2}+\sqrt{\frac{w}{\beta}})$, where the second inequality follows from the assumption that $S(\Lambda-\lambda, \frac{\Lambda}{2}+\sqrt{\frac{w}{\beta}})$ is above the cure $S(\lambda, \lambda+\sqrt{\frac{w}{\beta}}) = S(\lambda)$ in $\lambda \in [\frac{\Lambda}{2}, \Lambda]$. Therefore, there is no intersection between the curve $S(\lambda,\mu'')$ and $S(\lambda, \frac{\Lambda}{2}+\sqrt{\frac{w}{\beta}})$ if $\lambda \in [\frac{\Lambda}{2}, \Lambda]$. There may or may not exist intersections in the interval $\lambda \in [0, \frac{\Lambda}{2}]$. Therefore, $\mu'' > \frac{\Lambda}{2}+\sqrt{\frac{w}{\beta}}$ is not a best response, since a large service rate will cover a small market. $\square$

The model with price-dependent social interactions We consider the impact of price-dependent social interactions on the optimal price and service rate decisions for the monopolist. The social interaction intensity is denoted as $\alpha(p)$, which is assumed to be decreasing in p, i.e., the higher price of the service, the lower social interaction intensity. Consumer expected surpus is

$$S(\lambda, p, \mu) = v + \alpha(p)\lambda - CW(\lambda,\mu) - p$$

and the profit maximization problem of the monopolist is given as

$$\begin{aligned} \max_{\mu,p} \pi(\mu,p) \quad &= \quad \max_{\mu,p}(p-\beta\mu)\lambda \qquad (6.1.11)\\ s.t. \quad & \quad v + \alpha(p)\lambda - p - CW(\lambda,\mu) \geq 0 \\ & \quad \lambda \in [0,\Lambda],\ 0 \leq \lambda < \mu \end{aligned}$$

Since in equilibrium, consumer expected surplus is always reduced to the reservation level, the problem is reduced as

$$\begin{aligned} \max_{\mu,p} \pi(\mu,p) \quad &= \quad \max_{\mu,p}(v + \alpha(p)\lambda - CW(\lambda,\mu) - \beta\mu)\lambda \qquad (6.1.12)\\ s.t. \quad & \quad \lambda \in [0,\Lambda],\ 0 \leq \lambda < \mu \end{aligned}$$

for any given $\lambda \in [0,\Lambda]$, the optimal service rate will be $\mu = \lambda + \sqrt{\frac{w}{\beta}}$, and the above problem is reduced as

$$\max_{\lambda\in[0,\Lambda],p} (v + \alpha(p)\lambda - \beta\lambda - 2\sqrt{\beta w})\lambda$$

Since given $\lambda \in [0, \Lambda]$, and $\mu = \lambda + \sqrt{\frac{w}{\beta}}$, the optimal price satisfies $v + \alpha(p)\lambda - p - \sqrt{\beta w} = 0$, denoted as $p(\lambda)$, which is unique and increasing in λ. Therefore, the problem is reduced as

$$\max_{\lambda \in [0,\Lambda]} (v + \alpha(p(\lambda))\lambda - \beta\lambda - 2\sqrt{\beta w})\lambda$$

We define $F(\lambda) = \alpha(p(\lambda))$ for simplicity, with $n-th$ order derivative with respect to λ as $F^{(n)}$. It is straigthforward that $F^{(1)} \leq 0$. We assume $F^{(n)} \leq 0$, $n \geq 2$. The first and the second order conditions of the above profit maximizing problem are

$$\pi^{(1)}(\lambda) = v - 2\sqrt{\beta w} + 2F\lambda - 2\beta\lambda + F^{(1)}\lambda^2,\ \pi^{(2)}(\lambda) = 2F - 2\beta + 4F^{(1)}\lambda + F^{(2)}\lambda^2$$

and the third order derivative is $\pi^{(3)} = 6F^{(1)} + 6F^{(2)}\lambda + F^{(3)}\lambda^2 \leq 0$. Therefore, if $\pi^{(2)}(\Lambda) \geq 0$, the profit function is convex in $\lambda \in [0, \Lambda]$, i.e., the optimal arrival rate is $\lambda^m = \Lambda$; while if $\pi^{(2)}(\Lambda) < 0$, the optimal arrival rate will be

$$\lambda^m = \min(\lambda^\star, \Lambda)$$

where $\lambda^\star$ satisfies the first order condition $\pi^{(1)}(\lambda^\star) = 0$. Correspondingly, the optimal price and service rate will be $\mu^m = \lambda^m + \sqrt{\frac{w}{\beta}}$ and $p^m = p(\lambda^m)$.

6.2 Appendix II: Proofs in Chapter 3

Proof of Proposition 3.1

Proof. We have three situations: the steady state arrival rate under either policy is smaller than the initial arrival rate; the steady state arrival rate under either policy is larger than the initial arrival rate; the initial arrival rate in the middle of the two steady state arrival rates.

If $\lambda_0 \leq \lambda^c(\mu, p)$, the sequence $\lambda_1, \lambda_2, \cdots$, is increasing, we have

$$\begin{aligned} W(\lambda_0) &= (p - \beta\mu)\sum_{t=0}^{\infty}\delta^t\left(\mu - \frac{w}{v - p + \alpha\lambda_t}\right) \\ &\leq (p - \beta\mu)\frac{\lambda^c(\mu, p)}{1 - \delta} \leq \frac{\pi^c(\mu^{II}, p(\mu^{II}))}{1 - \delta} \end{aligned} \tag{6.2.1}$$

since the arrival rate is increasing. We have $\pi^c(\mu^{II}, p(\mu^{II})) \geq \pi^c(\mu^I, p(\mu^I)) \geq \pi_1(\mu^I, p(\mu^I))$, since the profit function is increasing in λ_0 and $\lambda_0 \leq \lambda^c(\mu, p)$. Therefore, if $\lambda_0 \leq \lambda_{\min}$, under either policy, the arrival rate path and the profit path are both increasing, indicating $\pi^c(\mu^{II}, p(\mu^{II}))$ is the largest single period profit. Therefore, we have $W(\lambda_0) \leq \frac{\pi^c(\mu^{II}, p(\mu^{II}))}{1-\delta}$.

While if $\lambda_0 > \lambda^c(\mu, p)$, the sequence $\lambda_1, \lambda_2, \cdots$, is decreasing, we have

$$\begin{aligned} W(\lambda_0) &= (p - \beta\mu) \sum_{t=0}^{\infty} \delta^t \left(\mu - \frac{w}{v - p + \alpha\lambda_t} \right) \\ &\leq (p - \beta\mu) \frac{\lambda_1}{1-\delta} \leq \frac{\pi_1(\mu^I, p(\mu^I))}{1-\delta} \end{aligned} \tag{6.2.2}$$

since the arrival rate and the profit path are both decreasing. We have $\pi^c(\mu^{II}, p(\mu^{II})) \geq \pi^c(\mu^I, p(\mu^I))$ and $\pi_1(\mu^I, p(\mu^I)) \geq \pi_1(\mu^{II}, p(\mu^{II}))$. Therefore, the two profit sequences $\{\pi_t^I, t = 1, 2, \cdots, \infty\}$ and $\{\pi_t^{II}, t = 1, 2, \cdots, \infty\}$ intersect exact once. If $\lambda_0 \geq \lambda_{\max}$, under either policy, the arrival rate path is decreasing, indicating $\pi_1(\mu^I, p(\mu^I))$ is the largest single period profit. Therefore, we have the result $W(\lambda_0) \leq \frac{\pi_1(\mu^I, p(\mu^I))}{1-\delta}$.

If λ_0 is in the middle as $\lambda_{\min} < \lambda_0 < \lambda_{\max}$, the profit sequence $\{\pi_t^I, t = 1, 2, \cdots, \infty\}$ must be decreasing while the profit sequence $\{\pi_t^{II}, t = 1, 2, \cdots, \infty\}$ must increase based on the intersection once property. Thus, we have $W(\lambda_0) \leq \max(\frac{\pi^c(\mu^{II}, p(\mu^{II}))}{1-\delta}, \frac{\pi_1(\mu^I, p(\mu^I))}{1-\delta})$. Therefore, the above results in the proposition hold. □

Proof of Lemma 3.2

Proof. The proof is based on Topkis' theorem (Topkis, 1998). We can also calculate the optimal arrival rate from the first order condition as

$$\pi_2 = \frac{\partial \pi(\lambda, \hat{\lambda})}{\partial \hat{\lambda}} = p - \beta \left(2\hat{\lambda} + \frac{w}{A + \alpha\lambda} \right) = 0 \Rightarrow \tilde{\lambda}(\lambda) = \frac{p}{2\beta} - \frac{w}{2(A + \alpha\lambda)}$$

which is increasing in λ. We have the optimal single period profit as $\pi^M(\lambda) = \frac{1}{4\beta} \left(p - \frac{\beta w}{A+\alpha\lambda} \right)^2$ which is increasing in λ. The single period profit maximizing service rate is

$$\tilde{\mu}(\lambda) = \tilde{\lambda}(\lambda) + \frac{w}{A + \alpha\lambda} = \frac{p}{2\beta} + \frac{w}{2(A + \alpha\lambda)}$$

which is decreasing in λ. □

Proof of Lemma 3.4

Proof. The single period profit function $\pi(\lambda, \hat{\lambda})$ is supermodular. For fixed price p, the constraint set of $\hat{\lambda}$ is bounded in the set $\Lambda(\lambda) = [0, \frac{p}{\beta} - \frac{w}{A+\alpha\lambda}]$ which is ascending in λ, i.e., for any $\lambda > \lambda'$ and $x \in \Lambda(\lambda)$, $x' \in \Lambda(\lambda')$, it follows that $\min(x, x') \in \Lambda(\lambda')$ and $\max(x, x') \in \Lambda(\lambda)$. Therefore, by Theorem 2.8.2 in Topkis (1998), $\hat{\lambda}^\star(\lambda) = \arg\max_{\hat{\lambda}}\{\pi(\lambda, \hat{\lambda}) + \delta V(\hat{\lambda})\}$ is increasing in λ. The monotonicity of the arrival rate path is proved by induction. If $\lambda_1^\star = \lambda^\star(\lambda_0) > \lambda_0$, we have $\lambda_2^\star = \lambda^\star(\lambda_1^\star) > \lambda^\star(\lambda_0) = \lambda_1^\star > \lambda_0$. Suppose $\lambda_t^\star > \lambda_{t-1}^\star$. We have $\lambda_{t+1}^\star = \lambda^\star(\lambda_t^\star) > \lambda^\star(\lambda_{t-1}^\star) = \lambda_t^\star$ indicating $\lambda_t^\star$ is increasing. While if $\lambda_1^\star = \lambda^\star(\lambda_0) < \lambda_0$, we will have a decreasing sequence $\{\lambda_t^\star, t \geq 1\}$. Therefore, the arrival rate path in the strategic policy is monotonic. □

Proof of Theorem 3.1

Proof. We first prove the solution of the steady state $\lambda^{\star\star}$ based on *Euler equation.* The partial derivatives of $\pi(\lambda, \hat{\lambda})$ with respect to λ and $\hat{\lambda}$ are defined as π_1 and π_2 respectively. If the steady state exists, we have $0 = \left(\pi_2(\lambda^{\star\star}, \hat{\lambda}) + \delta\pi_1(\hat{\lambda}, \lambda^{\star\star})\right)|_{\hat{\lambda}=\lambda^{\star\star}}$, or

$$0 = p - \beta\left(2\hat{\lambda} + \frac{w}{A + \alpha\lambda^{\star\star}}\right) + \delta\frac{\alpha\beta w\lambda^{\star\star}}{(A + \alpha\hat{\lambda})^2}|_{\hat{\lambda}=\lambda^{\star\star}}$$

Therefore, the steady state arrival rate $\lambda^{\star\star}$ is the solution to the following cubic function as $(p - 2\beta\lambda)(A + \alpha\lambda)^2 - \beta w(A + \alpha\lambda) + \delta\alpha\beta w\lambda = 0$.

We also have

$$\lim_{t\to\infty} \delta^t\pi_1(\lambda^{\star\star}, \lambda^{\star\star})\lambda^{\star\star} = \lim_{t\to\infty} \delta^t\frac{\alpha\beta w(\lambda^{\star\star})^2}{(A + \alpha\lambda^{\star\star})^2} = 0$$

since the arrival rate is bounded. Therefore, the transvasality condition is satisfied, indicating the solution to the above cubic function is a sufficient and necessary condition for the steady state (Stokey et al. 1989, Theorem 4.15).

We next prove the uniqueness of the solution in the interval $(0, \frac{p}{\beta})$. Define $F(\lambda) = (p - 2\beta\lambda)(A + \alpha\lambda)^2 - \beta w(A + \alpha\lambda) + \delta\alpha\beta w\lambda$. There exit three solutions, namely $\lambda^{(i)}$, $i = 1, 2, 3$, to the equation $F(\lambda) = 0$. We have $F(0) = pA^2 - \beta wA > 0$ based on the assumption, and $F(\frac{p}{\beta}) = -p(A + \frac{\alpha p}{\beta})^2 - \beta w(A + \frac{\alpha p}{\beta}) + \delta\alpha wp = -p(A + \frac{\alpha p}{\beta})^2 - \beta wA - (1 - \delta)\alpha wp < 0$. Since we also have $F(-\frac{A}{\alpha}) = -\delta\beta wA < 0$, $\forall\lambda \gg \frac{p}{\beta}, F(\lambda) < 0$, and $\forall\lambda \ll -\frac{A}{\alpha}, F(\lambda) > 0$, the three solutions satisfy $\lambda^{(1)} < -\frac{A}{\alpha} <$

$\lambda^{(2)} < 0 < \lambda^{(3)} < \frac{p}{\beta}$. We have the unique solution $\lambda^{\star\star} = \lambda^{(3)} \in (0, \frac{p}{\beta})$. Besides, the derivative of $F(\lambda)$ at $\lambda^{\star\star}$ is negative.

Suppose $\delta_1 < \delta_2$. Define $F(\lambda, \delta_i) = (p - 2\beta\lambda)(A + \alpha\lambda)^2 - \beta w(A + \alpha\lambda) + \delta_i \alpha\beta w\lambda$, and $\lambda^{\star\star}(\delta_i) = \arg(F(\lambda, \delta_i) = 0) \in (0, \frac{p}{\beta})$, $i = 1, 2$. We have $F(\lambda^{\star\star}(\delta_1), \delta_2) = (p - 2\beta\lambda^{\star\star}(\delta_1))(A + \alpha\lambda^{\star\star}(\delta_1))^2 - \beta w(A + \alpha\lambda^{\star\star}(\delta_1)) + \delta_2 \alpha\beta w \lambda^{\star\star}(\delta_1) > F(\lambda^{\star\star}(\delta_1), \delta_1) = 0$. Therefore, we have $0 < \lambda^{\star\star}(\delta_1) < \lambda^{\star\star}(\delta_2) < \mu$ based on the fact that $F(\lambda^{\star\star}(\delta_2), \delta_2) = 0$ and $\forall \lambda \in (0, \lambda^{\star\star}(\delta_2))$, $F(\lambda, \delta_2) > 0$. Therefore, $\lambda^{\star\star}(\delta)$ is increasing in δ. The monotonicity of $\lambda^{\star\star}(\alpha)$ in terms of α can be proved similarly.

We compare the value of $\lambda^{\star\star}$ in the strategic policy and λ^M in the myopic policy. From the previous result, we know $\lambda^M \in (0, \frac{p}{\beta})$ is the unique solution to the following equation $\lambda = \frac{p}{2\beta} - \frac{w}{2(A+\alpha\lambda)}$, which can be reformulated as $G(\lambda) = (p - 2\beta\lambda)(A + \alpha\lambda)^2 - \beta w(A + \alpha\lambda) = 0$, i.e., $G(\lambda^M) = 0$. We have $F(\lambda^M) = G(\lambda^M) + \delta\alpha\beta w\lambda^M > 0$. Therefore, we have $0 < \lambda^M < \lambda^{\star\star} < \mu$ based on $F(\lambda^{\star\star}) = 0$ and $\forall \lambda \in (0, \lambda^{\star\star}(\delta_2))$, $F(\lambda, \delta_2) > 0$. $\square$

Proof of Proposition 3.4

Proof. In the steady state (strategic policy or myopic policy), the profit function is the same as that in the static single period problem as

$$\pi(\lambda, \lambda) = \left(p - \beta\left(\lambda + \frac{w}{A + \alpha\lambda}\right)\right)\lambda, \ \lambda = \lambda^M \text{ or } \lambda^{\star\star}$$

The derivative at λ^M is larger than that at $\lambda^{\star\star}$ since

$$\frac{\partial \pi(\lambda^{\star\star}, \lambda^{\star\star})}{\partial \lambda} = (1 - \delta)\frac{\alpha\beta w\lambda^{\star\star}}{(A + \alpha\lambda^{\star\star})^2} > 0$$

indicating $\lambda^M \leq \lambda^{\star\star} \leq \lambda^\star$ and $\pi(\lambda^M, \lambda^M) \leq \pi(\lambda^{\star\star}, \lambda^{\star\star}) \leq \pi(\lambda^\star, \lambda^\star)$. $\square$

Proof of Lemma 3.5

Proof. The cross derivative of $\pi(\lambda, \hat{\lambda})$ is $\pi_{\lambda\hat{\lambda}} = \alpha > 0$ indicating the supermodularity. We can also calculate the first order derivative of $\hat{\lambda}$ as

$$\pi_2 = \frac{\partial \pi(\lambda, \hat{\lambda})}{\partial \hat{\lambda}} = B + \alpha\lambda - \frac{w\mu}{(\mu - \hat{\lambda})^2} = 0 \Rightarrow \tilde{\lambda}(\lambda) = \mu - \sqrt{\frac{w\mu}{B + \alpha\lambda}}$$

which is increasing λ. The price will be set as

$$\tilde{p}(\lambda) = v + \alpha\lambda - \frac{w}{\mu - \tilde{\lambda}(\lambda)} = v + \alpha\lambda - \sqrt{\frac{w(B+\alpha\lambda)}{\mu}}$$

with the first order derivative in terms of λ as $\frac{\partial \tilde{p}(\lambda)}{\partial \lambda} = \alpha - \frac{\alpha}{2}\sqrt{\frac{w}{\mu(B+\alpha\lambda)}}$ which is increasing in $\lambda \in (0, \mu)$, indicating the derivative is bounded in $(\alpha - \frac{\alpha}{2}\sqrt{\frac{w}{\mu B}}, \alpha - \frac{\alpha}{2}\sqrt{\frac{w}{\mu(B+\alpha\mu)}})$. Based on the assumption $\mu B > w$, we have $\alpha - \frac{\alpha}{2}\sqrt{\frac{w}{\mu B}} > 0$. Therefore, $\forall \lambda \in (0, \mu)$, $\frac{\partial \tilde{p}(\lambda)}{\partial \lambda} > 0$, in the myopic policy, the price is also increasing in λ. □

Proof of Lemma 3.7

Proof. λ_t^M is increasing in λ_{t-1}^M. Since $\tilde{\lambda}(\lambda^M) = \lambda^M$, if $\lambda_{t-1}^M > \lambda^M$, we have $\lambda_t^M > \lambda^M$. We also have

$$\lambda_t^M - \lambda_{t-1}^M = \left(\mu - \sqrt{\frac{w\mu}{B + \alpha\lambda_{t-1}^M}}\right) - \lambda_{t-1}^M < 0$$

based on the assumption, indicating $\lambda^M < \lambda_t^M < \lambda_{t-1}^M$. Correspondingly, $p^M = \tilde{p}(\lambda^M)$. We have $p_t^M = \tilde{p}(\lambda_{t-1}^M) \geq \tilde{p}(\lambda_t^M) = p_{t+1}^M$. The rest is by induction, which is omitted here. The case for $\lambda_{t-1}^M < \lambda^M$ can be similarly proved. □

Proof of Theorem 3.2

Proof. The proof is based on Euler equation. The partial derivatives of $\pi(\lambda, \hat{\lambda})$ with respect to λ and $\hat{\lambda}$ are $\pi_1 = \alpha\lambda_t$ and $\pi_2 = B + \alpha\lambda_{t-1} - \frac{w\mu}{(\mu-\lambda_t)^2}$ respectively. We have $0 = \left(\pi_2(\lambda^{\star\star}, \hat{\lambda}) + \delta\pi_1(\hat{\lambda}, \lambda^{\star\star})\right)|_{\hat{\lambda}=\lambda^{\star\star}}$, or

$$0 = \left(B + \alpha\lambda^{\star\star} - \frac{w\mu}{(\mu - \hat{\lambda})^2}\right) + \delta\alpha\lambda^{\star\star}|_{\lambda=\lambda^{\star\star}}$$

where $\hat{\lambda} = \lambda^{\star\star}$ is the solution to the cubic function $(B + (1+\delta)\alpha\lambda)(\mu - \lambda)^2 - w\mu = 0$. We also have the transevasility condition

$$\lim_{t\to\infty} \delta^t \pi_1(\lambda^{\star\star}, \lambda^{\star\star})\lambda^{\star\star} = \lim_{t\to\infty} \delta^t \alpha(\lambda^{\star\star})^2 = 0$$

since $\hat{\lambda}$ is bounded. Therefore, the solution to the above cubic function is a sufficient and necessary condition for the steady state.

We next prove the uniqueness of the solution in $(0, \mu)$. Define $F(\lambda) = (B + (1+\delta)\alpha\lambda)(\mu-\lambda)^2 - w\mu$. We have $F(0) = B\mu^2 - w\mu > 0$ based on the assumption, and $F(\mu) = -w\mu < 0$. Since it is a cubic function, there are three solutions, namely $\lambda^{(i)}, i = 1, 2, 3$ to the equation $F(\lambda) = 0$. We also have $\forall\lambda \gg \mu, F(\lambda) > 0$; $\forall\lambda \ll 0, F(\lambda) < 0$. Therefore, these three solutions satisfy $\lambda^{(1)} < 0 < \lambda^{(2)} < \mu < \lambda^{(3)}$. Therefore there exists a unique solution $\lambda^{\star\star} = \lambda^{(2)}$ in the interval $(0, \mu)$. Besides, the derivative of $F(\lambda)$ at $\lambda^{\star\star}$ is negative.

Suppose $\delta_1 < \delta_2$. Define $F(\lambda, \delta_i) = (B + (1+\delta_i)\alpha\lambda)(\mu-\lambda)^2 - w\mu$, and $\lambda^{\star\star}(\delta_i) = \arg\left(F(\lambda, \delta_i) = 0\right) \in (0, \mu)$, $i = 1, 2$. We have $F(\lambda^{\star\star}(\delta_1), \delta_2) = (B + (1+\delta_2)\alpha\lambda^{\star\star}(\delta_1))(\mu - \lambda^{\star\star}(\delta_1))^2 - w\mu > (B + (1+\delta_1)\alpha\lambda^{\star\star}(\delta_1))(\mu - \lambda^{\star\star}(\delta_1))^2 - w\mu = F(\lambda^{\star\star}(\delta_1), \delta_1) = 0$. Therefore, we have $0 < \lambda^{\star\star}(\delta_1) < \lambda^{\star\star}(\delta_2) < \mu$ based on the fact that $F(\lambda^{\star\star}(\delta_2), \delta_2) = 0$ and $\forall\lambda \in (0, \lambda^{\star\star}(\delta_2))$, $F(\lambda, \delta_2) > 0$. Therefore, $\lambda^{\star\star}(\delta)$ is increasing in δ. The monotonicity of $\lambda^{\star\star}(\alpha)$ in terms of α can be proved similarly.

We compare the value of $\lambda^{\star\star}$ in the strategic policy and λ^M in the myopic policy. From the previous result, we know $\lambda^M \in (0, \mu)$ is the unique solution to the following cubic function $G(\lambda) = (B + \alpha\lambda)(\mu - \lambda)^2 - w\mu$. We have $F(\lambda^M) = (B + (1+\delta)\alpha\lambda^M)(\mu - \lambda^M)^2 - w\mu > G(\lambda^M) = 0$, since $0 < \delta < 1$. Therefore, we have $0 < \lambda^M < \lambda^{\star\star} < \mu$ based on $F(\lambda^{\star\star}) = 0$ and $\forall\lambda \in (0, \lambda^{\star\star}(\delta_2))$, $F(\lambda, \delta_2) > 0$. □

Proof of Theorem 3.3

Proof. For three-period problem, using backward-induction, after simplification, we have the following relationships

$$\mu_3^\star(\lambda_2^\star) = \mu_3^M(\lambda_2^M) + \frac{\alpha}{2\beta}\left(\frac{\alpha}{2\beta}\frac{\delta\alpha}{4\beta} + \frac{\delta\alpha}{4\beta}\right),\ p_3^\star(\lambda_2^\star) = p_3^M(\lambda_2^M) + \alpha\left(\frac{\alpha}{2\beta}\frac{\delta\alpha}{4\beta} + \frac{\delta\alpha}{4\beta}\right),$$

$$\lambda_3^\star(\lambda_2^\star) = \lambda_3^M(\lambda_2^M) + \frac{\alpha}{2\beta}\left(\frac{\alpha}{2\beta}\frac{\delta\alpha}{4\beta} + \frac{\delta\alpha}{4\beta}\right),$$

$$\mu_2^\star(\lambda_1^\star) = \mu_2^M(\lambda_1^M) + \frac{\alpha}{2\beta}\frac{\delta\alpha}{4\beta} + \frac{\delta\alpha}{4\beta},\ p_2^\star(\lambda_1^\star) = p_2^M(\lambda_1^M) + \alpha\frac{\delta\alpha}{4\beta},$$

$$\lambda_2^\star(\lambda_1^\star) = \lambda_2^M(\lambda_1^M) + \frac{\alpha}{2\beta}\frac{\delta\alpha}{4\beta} + \frac{\delta\alpha}{4\beta}$$

$$\mu_1^\star(\lambda_0) = \mu_1^M(\lambda_0) + \frac{\delta\alpha}{4\beta},\ p_1^\star(\lambda_0) = p_1^M(\lambda_0),\ \lambda_1^\star(\lambda_0) = \lambda_1^M(\lambda_0) + \frac{\delta\alpha}{4\beta}$$

Therefore, we conjecture that the service rate path, price path and the arrival rate path follow the above relationship. Clearly, the above relationship is valid for the problem with period 2 and 3. Suppose the above relationship is valid for the finite-horizon problem with n periods. We can use induction to prove the above relationship is also valid for the finite-horizon problem with $n+1$ periods. Therefore, in the problem with $n+1$ periods, at stage $t = n-1$, we have the arrival rate relationship $\lambda_{n-1}^\star = \lambda_{n-1}^M + \frac{\delta\alpha}{4\beta}\left(\frac{1-(\frac{\alpha}{2\beta})^{n-1}}{1-\frac{\alpha}{2\beta}}\right)$. In period $n+1$, the service provider adopts the myopic policy given λ_n, where we can solve the optimal service rate, price and arrival rate in period $n+1$ as $\mu_{n+1}^\star(\lambda_n),\ p_{n+1}^\star(\lambda_n),\ \lambda_{n+1}^\star(\lambda_n)$. The profit function is $\pi(\lambda_n)$ which is linear in λ_n. Then in period n, given the arrival rate $\lambda_{n-1}^\star$, we solve the two-period profit optimization problem with the optimal service rate, price and arrival rate in period n as $\mu_n^\star(\lambda_{n-1}^\star),\ p_n^\star(\lambda_{n-1}^\star),\ \lambda_n^\star(\lambda_{n-1}^\star)$. Respectively, we can solve the service rate, price and the arrival rate in the myopic policy in period n and $n+1$, given λ_{n-1}^M. Substituting $\lambda_{n-1}^\star = \lambda_{n-1}^M + \frac{\delta\alpha}{4\beta}\left(\frac{1-(\frac{\alpha}{2\beta})^{n-1}}{1-\frac{\alpha}{2\beta}}\right)$, we can conclude that $\mu_n^\star(\lambda_{n-1}^\star),\ p_n^\star(\lambda_{n-1}^\star),\ \lambda_n^\star(\lambda_{n-1}^\star)$ in period n follow the above relationship. Substituting $\lambda_n^\star$ into $\mu_{n+1}^\star(\lambda_n),\ p_{n+1}^\star(\lambda_n),\ \lambda_{n+1}^\star(\lambda_n)$, we can see the above relationship is also valid. Therefore, we conclude the above relationships are valid for all $N \geq 2$. □

6.3 Appendix III: Proofs in Chapter 4

Proof of Proposition 4.1

Proof. Suppose $W_t(q^\star)$ as the optimal long-run discounted profit from period t onwards with the service effort decision $q^\star$ in period t. Given the service effort decisions $q_0, q_1, \cdots, q_t$ from the initial period to period t, the cumulative profit from the period $t+1$ onwards only depends on the service effort decision from period $t+1$ onwards. Therefore, if $q^\star$ is the optimal decision in period t for the total profit from period t onwards, then $q^\star$ will be also the optimal decision in period $t+1$ for the total profit from period $t+1$ onwards. Therefore, we have $W_t(q^\star) = \max_{q\in[q_L,q_H]} \pi(x) + \delta\left((\kappa+\tau)F(x) - \tau + 1\right) W_{t+1}(q^\star)$, where the optimal decision must satisfy $x = q^\star$. Therefore, we have $\pi'(q^\star) + \delta(\kappa+\tau)F_1(q^\star)W(q^\star) = 0$, which indicates $q^\star \geq q^m$, since

$\pi'(q^\star) < 0$. We also have $W(q^\star) = \pi(q^\star) + \delta\left((\kappa+\tau)F(q^\star) - \tau + 1\right)W(q^\star)$, which indicates $W(q^\star) = \frac{\pi(q^\star)}{1-\delta((\kappa+\tau)F(q^\star)-\tau+1)}$. Therefore, the optimal service effort policy is a constant policy in each period. Since q^c maximizes $W(q)$, the optimal service effort policy will be a constant policy with $q_t = q^c$ in each period. At $q^\star = q^c$, we have $\pi'(q^c) = -\frac{\delta(\kappa+\tau)F_1(q^c)\pi(q^c)}{1-\delta((\kappa+\tau)F(q^c)-\tau+1)}$, where the RHS is decreasing in κ. Therefore, q^c is increasing in κ. The RHS is also decreasing in τ, since the derivative in terms of τ is $\frac{-\delta F_1(q^c)\pi(q^c)(1-\delta\kappa)}{[1-\delta((\kappa+\tau)F(q^c)-\tau+1)]^2} \leq 0$ based on the assumption that $1 > \delta(\kappa+1)$. Therefore, q^c is increasing in κ and τ. Clearly, $\frac{\pi(q^m)}{1-\delta((\kappa+\tau)F(q^m)-\tau+1)}$ is the myopic policy, where the service effort decision is only to maximize each single period profit, and $\frac{\pi(q^m)}{1-\delta(\kappa+1)}$ is the maximum long-run profit if the all served consumers are satisfied and the single-period profit is the largest. We have $\frac{\pi(q^m)}{1-\delta((\kappa+\tau)F(q^m)-\tau+1)} \leq W(q^c) \leq \frac{\pi(q^m)}{1-\delta(\kappa+1)}$. □

Proof of Proposition 4.2

Proof. Given the overall experience $r' = \gamma r + (1-\gamma)q$, we reformulate the single period profit as $v(r, r') = \pi(\frac{r'-\gamma r}{1-\gamma})$ since $q = \frac{r'-\gamma r}{1-\gamma}$. Therefore, the problem is reformulated as a sequence problem in terms of overall experience

$$V(r_t) = \max_{r_{t+1}\in R_{t+1}} v(r_t, r_{t+1}) + \delta H\left(\frac{r_{t+1}-\gamma r_t}{1-\gamma}, r_t\right) V(r_{t+1}) \tag{6.3.1}$$

where the firm determines the optimal overall experience $r_{t+1} \in R_{t+1} = [\gamma r_t + (1-\gamma)q_L, \gamma r_t + (1-\gamma)q_H]$ through service effort decision $q_t \in [q_L, q_H]$. If the problem admits a steady state service effort level $q^{\star\star}$, based on the Euler equation, we have the first order derivative of the RHS at r_{t+1} as

$$\left\{v(r_t, r_{t+1}) + \delta H\left(\frac{r_{t+1}-\gamma r_t}{1-\gamma}, r_t\right)\left(v(r_{t+1}, r_{t+2}) + \delta H\left(\frac{r_{t+2}-\gamma r_{t+1}}{1-\gamma}, r_{t+1}\right) U(r_{t+2})\right)\right\}$$

where at the steady state, $r_t = r_{t+1} = r_{t+2} = q^{\star\star}$, and the above first order derivative equals to zero, simplified as $(1-\gamma\delta H)\pi' + U\left[\delta H_1 + \delta^2 H\left(-\gamma H_1 + (1-\gamma)H_2\right)\right] = 0$, or

$$\frac{\pi'}{U} + \frac{\delta H_1 + \delta^2 H\left(-\gamma H_1 + (1-\gamma)H_2\right)}{1-\gamma\delta H} = 0 \tag{6.3.2}$$

where $H = H(q^{\star\star}, q^{\star\star})$, $H_1 = H_1(q^{\star\star}, q^{\star\star})$, $H_2 = H_2(q^{\star\star}, q^{\star\star})$, $\pi' = \pi'(q^{\star\star})$, $U = U(q^{\star\star})$. The above equation is denoted as *the steady state condition.*

Since $\delta H < 1$, $H_2 < 0$ and $H_1 + H_2 \geq 0$, $\delta H_1 + \delta^2 H\left(-\gamma H_1 + (1-\gamma)H_2\right) \geq \delta H_1 + \delta\left(H_1 - \gamma H_1 + (1-\gamma)H_2 - H_1\right) = (1-\gamma)(H_1 + H_2) \geq 0$. Therefore, we have $\pi' < 0$, which indicates $q^{\star\star} \geq q^m$.

Recall that at q^c, we have $\frac{\pi'(q)}{U(q)} + \delta\left(H_1(q,q) + H_2(q,q)\right)|_{q=q^c} = 0$. At q^c, the first order derivative of $\frac{\pi'(q^c)}{U(q^c)}$ is $\frac{\pi''U - \pi'U'}{U^2}|_{q=q^c} = \frac{\pi''U}{U^2}|_{q=q^c} \leq 0$, indicating $\frac{\pi'}{U}$ is decreasing at q^c. From the steady state condition, the second term can be simplified as

$$\delta\left((H_1 + H_2) - \frac{(1-\delta H)}{(1-\gamma\delta H)}H_2\right) \geq \delta(H_1 + H_2)$$

which indicates $\frac{\pi'}{U}|_{q=q^{\star\star}} \leq \frac{\pi'(q^c)}{U(q^c)}$. Therefore, we have $q^{\star\star} \geq q^c$. □

Proof of Corollary 4.1

Proof. Clearly, the second term in the LHS of the steady state condition is increasing in δ, which indicates $\frac{\pi'}{U}$ will decrease if δ increases. Therefore, $q^{\star\star}$ is increasing in δ. Rearrange the second term in the LHS, we have $\delta\left((H_1 + H_2) - \frac{(1-\delta H)}{(1-\gamma\delta H)}H_2\right)$, which is increasing in γ, indicating $q^{\star\star}$ is increasing in γ.

Substituting $U = \frac{\pi}{1-\delta H}$ into the steady state condition, we have

$$\pi' + \frac{\delta\pi(H_1 + H_2)}{1-\delta H} - \frac{\delta\pi H_2}{1-\gamma\delta H} = 0 \tag{6.3.3}$$

where the term $\frac{\delta\pi(H_1+H_2)}{1-\delta H}$ is increasing in κ, and $-\frac{\delta\pi H_2}{(1-\gamma\delta H)}$ is also increasing in κ. Therefore, π' is decreasing in κ, indicating the steady state $q^{\star\star}$ is increasing in κ, since π' is decreasing at $q^{\star\star} \geq q^m$.

We have the first order derivative of $\frac{\delta\pi(H_1+H_2)}{1-\delta H}$ in terms of τ as

$$\delta\pi\frac{(F_1 + F_2)(1-\delta H) + (H_1 + H_2)\delta(F-1)}{(1-\delta H)^2} = \delta\pi\frac{(F_1 + F_2)(1-\delta\kappa)}{(1-\delta H)^2} \geq 0$$

and the first order derivative of $\frac{\delta\pi H_2}{(1-\gamma\delta H)}$ in terms of τ is

$$\delta\pi\frac{F_2(1-\gamma\delta H) + H_2\gamma\delta(F-1)}{(1-\gamma\delta H)^2} = \delta\pi\frac{F_2(1-\gamma\delta\kappa)}{(1-\gamma\delta H)^2} \leq 0$$

which indicates π' is decreasing in τ, indicating the steady state $q^{\star\star}$ is also increasing in τ. □

Proof of Lemma 4.1

Proof. Since $\pi(q)$ is concave, the reformulated single period profit function $v(r,r')$ is supermodular, since the cross derivative $v_{12}(r,r') = -\frac{\gamma}{(1-\gamma)^2}\pi''(\frac{r'-\gamma r}{1-\gamma}) \geq 0$. The cross partial derivative of $\delta\left((\kappa+\tau)F\left(\frac{r'-\gamma r}{1-\gamma}, r\right) - \tau\right)V(r')$ can be checked to be positive, based on the assumption that $F(q,r)$ is supermodular. Therefore, the derivative of the RHS of the reformulated sequence problem in terms of r_{t+1} is supermodular, which indicates the optimal state path in period $t+1$, termed as $r^\star_{t+1} = s^\star(r_t)$ is increasing in r_t based on the supermodularity property (Topkis, 1998). Suppose the optimal state path is not monotonic with any three sequential states as $r^\star_t < r^\star_{t+1} > r^\star_{t+2}$. Since $r^\star_{t+2} = s^\star(r^\star_{t+1}) \geq s^\star(r^\star_t) = r^\star_{t+1}$, which indicates a contradiction. Other cases can be similarly proved by contradiction. Therefore, the optimal state path $r^\star_t$ must monotonically converges to a steady state. Since the state path is monotonic and the domain $R_{t+1} = [\gamma r_t + (1-\gamma)q_L, \gamma r_t + (1-\gamma)q_H]$ is ascending in r_t and compact, there exists a unique steady state of overall experience path. The steady state condition also guarantees the uniqueness of the steady state, since π' is decreasing, and $\frac{\delta\pi(H_1+H_2)}{1-\delta H}$ is decreasing in q by the DGFR assumption, and the third term is also decreasing in q since H_2 is increasing in q due to the convexity of $F(q,r)$ on r. Therefore, the steady state is unique which is $q^{\star\star}$ in the previous result. □

Proof of Proposition 4.4

Proof. The single period profit is reformulated as $\hat{\Pi}(R,r) = \Pi(\frac{R-\gamma r}{1-\gamma}, r)$ where $R \in [\gamma r + (1-\gamma)q_L, \gamma r + (1-\gamma)q_H]$. The long-run discounted profit optimization problem is reformulated as

$$V(r_t) = \max_{r_{t+1}\in[\gamma r_t, 1]} \hat{\Pi}(r_{t+1}, r_t) + \delta\left((\kappa+\tau)F(\frac{r_{t+1}-\gamma r_t}{1-\gamma}, r_t) - \tau\right)V(r_{t+1}) \quad (6.3.4)$$

Suppose the steady state service effort level exists as $q^{\star\star}$. In the steady state, we have $q^\star(q^{\star\star}) = q^{\star\star}$. Applying Euler's equation, we have *the steady state condition*

$$(1-\gamma\delta H)\Pi_1 + (1-\gamma)\delta H\Pi_2 + \delta U\left[(1-\gamma\delta H)H_1 + (1-\gamma)\delta H H_2\right] = 0 \quad (6.3.5)$$

Substitute Π_1 and Π_2 into the above equation, we have

$$\begin{aligned}\pi'(q) &= \frac{-\delta U\left[(1-\gamma\delta H)H_1 + (1-\gamma)\delta H H_2\right] - (1-\delta H)R'(0)}{1-\gamma\delta H} \\ &= -\delta U\left[H_1 + \frac{(1-\gamma)\delta H}{1-\gamma\delta H}H_2\right] - \frac{(1-\delta H)R'(0)}{1-\gamma\delta H} \leq 0 \qquad (6.3.6)\end{aligned}$$

which is denoted as the steady state condition. Since $H_1 + \frac{(1-\gamma)\delta H}{1-\gamma\delta H}H_2 \geq H_1 + H_2$, $R'(0) \geq 0$. Therefore, the steady state (if exists) satisfies $q^{\star\star} \geq q^c \geq q^{M\star}$. □

Proof of Proposition 4.8

Proof. In the first case $\max(q_l^{\star\star}, q_h^{\star\star}) \leq r$, the condition says if the firm first decides a lower enough service effort q, such that $\gamma r + (1-\gamma)q \leq q_h^\star$, then the total discounted profit is less than the total discounted profit under the constant service effort policy, since if $\max(q_l^{\star\star}, q_h^{\star\star}) \leq r$, for any $q \leq r$,

$$\begin{aligned}U^h(r) &\geq \pi(q) + \delta\left((\kappa_l + \tau_h)F(q,r) - \tau_h\right)V^h(\gamma r + (1-\gamma)q) \\ &\geq \max_q \pi(q) + \delta\left((\kappa_l + \tau_h)F(q,r) - \tau_h\right)V^l(\gamma r + (1-\gamma)q) \\ &= V^l(r) \qquad (6.3.7)\end{aligned}$$

based on the previous result that the optimal state path of $V^l(r)$ is monotonically decreasing.

In the second case $q_h^{\star\star} \leq r \leq q_l^{\star\star}$, the condition says if the firm first decides a higher enough service effort q, such that $\gamma r + (1-\gamma)q \geq q_l^\star$, then the total discounted profit is less than the total discounted profit under the constant service effort policy, since if $q_h^{\star\star} \leq r \leq q_l^{\star\star}$, for any $q \geq r$,

$$\begin{aligned}U^h(r) &\geq \pi(q) + \delta\left((\kappa_h + \tau_l)F(q,r) - \tau_l\right)V^l(\gamma r + (1-\gamma)q) \\ &\geq \max_q \pi(q) + \delta\left((\kappa_h + \tau_r)F(q,r) - \tau_r\right)V^l(\gamma r + (1-\gamma)q) \\ &\geq V^l(r) \qquad (6.3.8)\end{aligned}$$

□